MATHEMATICAL METHODS - I

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Preamble : **To teach the meaning and significance of elementary mathematical tools in students of economics. To enable the students to understand the concepts and methods of mathematical techniques.**

Unit - I Number System

Numbers System – Real – Imaginary and Complex Numbers, Rational and Irrational Numbers – Graphs – Application of Graphs in Economics. **(17L)**

Unit - II Set Theory

Meaning – Types of sets – Set operations – Venn diagram – Cartesian Products – Functional Relations and Functions. **(18L)**

Unit - III Functions And Equations

Functions – Types and application of functions in Economics – Equations – Types of equations (Linear, Simultaneous Quadratic and Polynomial) Solving linear and Quadratic Equations – Application of equations in Economics. **(18L)**

Unit - IV Analytical Geometry

Distance between two Point in a Plane – Slope of Straight line. Different types of equations of a Straight line – Intersection of two lines – Perpendicular lines, Parallel – Application of Straight lines in Economics. **(19L)**

Unit – V Commercial Arithmetic

Percentage – Ratio and Proportion – Simple Interest – Annuities – Depreciation – Discounts – Banker's Discount – True Discount. **(18L)**

(Total: 90L)

Text Books:

1. **Mathematical Methods- Dr.Rathina Pandian**
2. **Mathematical Methods- Dr.Bose**

Reference Books:

1. **Mathematics for Economics – Metha – Madnani**
2. **Mathematics for Economics and Business – R.s. Bharathwaj**
3. **Mathematical Analysis for Economics - RGD.Allen**

CHAPTER - I

NUMBER SYSTEM

I Natural Number System

The set of natural number system is denoted as N. The numbers1, 2, 3, 4, ... etc are considered as natural numbers. It can be represented as

N = {1, 2, 3, 4, 5, ...}

According to Kronecker, a German mathematician, the Natural numbers are God's gift to mankind.

Integers

The numbers without fraction are called integers. (Eg) 1, 2, 3, 4, 5, 6 .... etc. The numbers like 2.5, ¼, 2½ etc. are not considered as integers.

The integers may either be positive or negative. There is a corresponding negative integer for every positive integer.

(Eg) 1, 2, 3, 4, 5, 6 .... etc are positive integers. For these positive integers there are corresponding negative integers.

i.e., -1, -2, -3, -4, -5, -6 .... etc.

The sum of two integers; subtraction of two integers; and multiplication of two integers are integers.

Whole Numbers

The union of the set of natural numbers and zero are called the set of whole numbers.

Rational Numbers

Any number in the form *m*/*n* where *m* and *n* are integers and $n \neq 0$ is called a rational number. Set of rational numbers is denoted by Q.

Characterisitics of Rational Numbers

1. Every integers can be represented in the form of *m*/*n*. Hence, the set of all integers is a subset of rational numbers.

2. The important character of rational numbers is that between any two rational numbers there is always at least one other rational number $\left(\text{i.e., } \frac{m+n}{2}\right)$

3. The rational numbers are of three types (i) **Proper fraction** (ii) **Improper fraction** and (iii) **Mixed fraction.** If $m < n$, it is called proper fraction (Eg:4/7). If $m > n$, it is called improper fraction (Eg:9/4). If an improper fraction is expressed in the form of a whole number and a proper fraction, it is called a mixed fraction.

(Eg) Representation of $\frac{9}{4}$ as $2\frac{1}{4}$

4. Every rational number can be expressed in decimal form. The decimal may either be **pure decimal** or **mixed decimal**

(Eg) $\frac{3}{4} = 0.75$ is a pure decimal; $\frac{4}{3} = 1.33$ is a mixed decimal.

5. When the rational number is expressed in the form of decimal it may either be a **Terminating decimal** or **Non-terminating decimal.**

(Eg) $\frac{3}{25} = 0.12$ is a terminating decimal; $\frac{7}{11} = 0.636363$ .... is a non-terminating decimal.

6. The sum of two rational numbers; the difference of two rational numbers; and the product of two rational numbers are rational numbers only.

Irrational Numbers

A number corresponding to a non-repeating infinite decimal is called Irrational Number (Eg) π, e, $\sqrt{5}$, $\sqrt{3}$, $\sqrt[3]{8}$ etc.

Real Numbers

The set of all rational and irrational numbers is called the set of Real Numbers.

Imaginary Numbers

The numbers in the form of $\sqrt{-9}$, $\sqrt{-6}$, $\sqrt{-3}$ etc are called Imaginary numbers.

Even Numbers

The natural numbers which are divisible by 2 are called Even Numbers.

Odd Numbers

All natural numbers which are not divisible by 2 are called Odd Numbers.

Prime Numbers

Prime Numbers are natural numbers except 1 which are not divisible by any natural number except 1 and itself.

(Eg) 2, 3, 5, 7 etc.

Co - prime Numbers:

Two prime numbers are called Co - prime number

(Eg) 2 and 3; 5 and 11 etc.

Complex Numbers

The numbers in the form $(a + ib)$ or $(a - ib)$ is called complex numbers, where a and b are real numbers and $i = \sqrt{-1}$. If $b = 0$, then the complex number is called 'Purely real complex numbers' and if $a = 0$ then the complex number is called 'Purely unreal complex numbers'

II HIGHEST COMMON FACTOR

Highest common factor is also called Greatest Common Factor or Greatest Common Divisor or Greatest Common Measure or Highest Common Divisor.

When X and Y are two integers and X is divided by Y, then X is called the '*dividend*' and Y is called the '*divisor*'. If there is no remainder when X is divided by Y, then Y is called the '*factor of* X'. If Y divides two or more integers without a remainder then Y is called '*common factor*' for them.

Highest common factor is the greatest integer which divides two or more integers.

(Eg) 16 can be divided by 1, 2, 4, 8, 16

24 can be divided by 1, 2, 3, 4, 6, 8, 12, 24

Hence, the highest integer which divides 16 and 24 is 8. Hence, 8 is called the Highest Common factor for 16 and 24.

Methods of calculating Highest Common Factor

The following are the methods of calculating HCF

1. Factor Method and 2. Division Method.

1. Factor Method

The following steps should be adopted to find out HCF under factor method.

(i) The Prime factors for each of the given integers should be identified

(ii) Identify all prime factors which are common to all given integers.

(iii) Find out the product of all common prime factors. This is called the Highest Common Factor.

Example: Find the HCF of (i) 1000 and 400 (ii) 3087 and 1323

(i) Prime factors of 1000 = $2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^3 \times 5^3$
Prime factors of 400 = $2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^4 \times 5^2$
The common prime factor = 2^3 and 5^2
$\therefore$ HCF = $2^3 \times 5^2 = 200$

(ii) Prime factors of 3087 = $3 \times 3 \times 7 \times 7 \times 7 = 3^2 \times 7^3$
Prime factors of 1323 = $3 \times 3 \times 3 \times 7 \times 7 = 3^3 \times 7^2$
The common prime factors = 3^2 and 7^2
$\therefore$ HCF = $3^2 \times 7^2 = 441$

2. Division Method

The following steps should be adopted to find out the HCF, un division method

(i) The larger integer should be divided by the smaller integer.
(ii) Divide the divisor of the first step with the remainder.
(iii) This process should be repeated until the remainder is zero.
(iv) The HCF is the last non-zero remainder.

Example 1. Find the HCF for (i) 8575 and 6125 (ii) 861 and 1476 a (iii) 471, 1256 and 3454

Solution

(i)
```
6125 ) 8575 ( 1
       6125
       ----
       2450 ) 6125 ( 2
              4900
              ----
              1225 ) 2450 ( 2
                     2450
                     ----
                     0000
```

The last non-zero remainder is the HCF. Hence HCF is 122

(ii)
```
861 ) 1476 ( 1
      861
      ---
      615 ) 861 ( 1
            615
            ---
            246 ) 615 ( 2
                  492
                  ---
                  123 ) 246 ( 2
                        246
                        ---
                        000
```

$\therefore$ HCF = 123

(iii) **Integers 3454 and 1256 are selected first**

```
1256 ) 3454 ( 2
       2512
       ----
        942 ) 1256 ( 1
               942
               ---
               314 ) 942 ( 3
                     942
                     ---
                     000
```

Divide 471 by 314

```
314 ) 471 ( 1
      314
      ---
      157 ) 314 ( 2
            314
            ---
            000
```

Hence, the HCF is 157

LEAST COMMON MULTIPLE

Least Common Multiple is otherwise called as Lowest Common Multiple. A number which is exactly divisible by two or more integers is called Least Common Multiple.

(Eg) If 2, 4, 5, are the three numbers, then the number 20 is exactly divisible by 2, 4 and 5. The number 20 is called the least common multiple.

There are two methods for calculating the Least Common Multiple.. They are, 1. Factors Method and 2. Short Division Method

1. Factors Method

In this method LCM is calculated through the prime factors.

Example: Calculate Least Common Multiple for 72, 324 and 432.

Solution:

Prime factors of 72 $= 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

Prime factors of 324 $= 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 2^2 \times 3^4$

Prime factors of 432 $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^4 \times 3^3$

LCM of 72, 324 and 432 $= 2^4 \times 3^4 = 16 \times 81 = 1296$

2. Short Division Method

In this method the LCM is calculated through dividing the integers with common factors.

Example: 1. Calculate Least Common Multiple for 225, 2025 and 3375

Solution:

5	225,	2025,	3375
5	45,	405,	675
5	9,	81,	135
3	9,	81,	27
3	3,	27,	9
3	1,	9,	3
	1,	3,	1

$$LCM = 5 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3 = 10125$$

Example: 2 The product of two numbers is 589824 and their HCF is 64. Find the possible pairs of the numbers.

Solution:

Let the prime factors of the two numbers be x and y respectively.

$$64x \,.\, 64y = 589824$$

$$4096xy = 589824$$

$$xy = \frac{589824}{4096} = 144$$

Hence, the possible pairs of the numbers are (1,144), (2,72), (4,36), (8,18), (16,9), (48,3)

Example: 3 The HCF of two numbers is 108 and their LCM is 2268. If one of the number is 756, find the other number.

Solution:

HCF x LCM of two numbers is the product of the two numbers.

i.e., $108 \times 2268 = 244944$

If one number is 756, the other number is $\frac{244922}{756} = 324$

Example: 4 Find the LCM of $\frac{3}{5}, \frac{2}{7}, \frac{6}{11}$

Solution:

$$\text{LCM of } 3, 2, 6 = \frac{\text{LCM of } 3, 2, 6}{\text{HCF of } 5, 7, 11}$$

$$= \frac{6}{1} = 6$$

Example: 5 Find the HCF of $\frac{6}{7}, \frac{3}{5}, \frac{9}{10}$

Solution:

$$\text{HCF of } \frac{6}{7}, \frac{3}{5}, \frac{9}{10} = \frac{\text{HCF of 6, 3, 9}}{\text{LCM of 7 5 10}}$$

$$= \frac{3}{70}$$

EXERCISE

I Choose the correct answer for the following

1. The numbers without fraction is called
   (i) Real Numbers (ii) Irrational Numbers (iii) Integers
   (iv) Prime Numbers

2. $\frac{3}{7}$ is a
   (i) Rational Number (ii) Index (iii) Imaginary Number
   (iv) Odd Number

3. The numbers in the form $\sqrt{-5}, \sqrt{-4}$ etc are called
   (i) Irrational Numbers (ii) Imaginary Numbers
   (iii) Integers (iv) Real Number

4. The natural numbers which are divisible by 2 is called
   (i) Odd Numbers (ii) Integers
   (iii) Even Numbers (iv) Irrational Numbers

5. A number which is not divisible by any natural numbers except 1 and itself is called
   (i) Prime Number (ii) Odd Number
   (iii) Even Numbers (iv) Irrational Numbers

6. The greatest integer which divide two or more integers is called
   (i) HCF (ii) LCM (iii) Prime Number
   (iv) Imaginary Numbers

7. A number which is exactly divisible by two or more integers is called
   (i) Indices (ii) Rational Numbers (iii) LCM (iv) HCF

II 1. Define Rational Numbers. State the characteristics of Rational Numbers.

2. What is Highest Common Factor? Briefly explain the different methods of calculating Highest Common Factor.

3. Define Least Common Multiple. Briefly explain the different methods of calculating Least Common Multiple.

4. Write short notes for the following.

   (i) Prime Numbers (ii) Real Numbers (iii) Rational Numbers
   (iv) Irrational Numbers (v) HCF (vi) LCM

III 1. Find the HCF for the following by Factor Method

   (i) 4500 and 32400 (ii) 3969 and 64827
   (iii) 972 and 5184 (iv) 1125 and 2025

2. Find the HCF for the following by Division Method

   (i) 758 and 970 (ii) 2745 and 1159
   (iii) 7938, 6426 and 5112 (iv) 2525, 2550, 2575

3. Calculate LCM by Factors Method and Short Division Method for the following

   (i) 5000 and 8000 (ii) 2916 and 5832
   (iii) 108, 216 and 972 (iv) 45, 75 and 375

4. The product of two number is 388800 and their HCF is 180. Find the possible pairs of the numbers

5. The HCF of two numbers is 420 and their LCM is 6300. If one of the number is 2100, find the other number.

6. Find the LCM of $\frac{5}{6}, \frac{2}{3}, \frac{8}{9}$

7. Find the HCF of $\frac{15}{7}, \frac{3}{8}, \frac{9}{14}$

8. What is the rational number of $\sqrt{3}$, $1/2$ $\sqrt{2}$ - $\sqrt{7}$,

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ANSWERS

I 1. (iii) 2. (i) 3. (ii) 4. (iii) 5. (i) 6. (i) 7. (iii)

III 1. (i) 900 (ii) 1323 (iii) 324 (iv) 225

2. (i) 2 (ii) 61 (iii) 18 (iv) 25

3. (i) 40,000 (ii) 17496 (iii) 1944 (iv) 1125

4. (1,12) (2,6) (3,4) 5. 1260

6. $\frac{40}{3}$ 7. $\frac{3}{56}$ 8. $1/2$

SET THEORY

Introduction

Set theory plays a vital role in all branches of Modern Mathematics. Set theory was first considered by George Contor (1845–1918).

MEANING

A set is a collection, assemblage, a group or a class of definite and well distinguished objects.

DEFINITION

A set is a collection of objects, each of which is associated with a definite property, say P, such that only those objects belong to the set which satisfy the property P.

Examples:

1. a pack of 52 cards
2. a team of hockey players
3. the students in a class
4. the days in the week
5. the books in the library
6. the flock of sheep
7. the set of integers 1,2,3,4,5,8,10

Thus the word set is used to denote collection, class family, flock, bunch, crowd, association etc.

Elements

The objects in a set are called as "Element's" or "Members" of the set.

Notation

It is convention to denote sets by capital letters A,B,C . . . . X,Y,Z. etc, and Elements by small letters a,b,c. . . x,y,z. etc.

If 'x' is an element of a Set 'A', then we denote it as $x \in A$, and if 'y' is not an element of the set 'A' then we denote it as $y \notin A$.

We use the symbol $\in$ (Epsilon) to indicate that x is "an element of" or x "belongs to "or x "is a member of " or x is "is contained in".

Example :

A = {1, 3, 5, 7} is a Set of the odd numbers 1, 3, 5, 7 which are elements of set A. Then, for element 3, we write $3 \in A = \{1, 3, 5, 7\}$.

The elements of the sets are separated by commas and written in Braces, or Curly Brackets, or Brace Brackets or Flowered Brackets i.e.,{ }.

An element is never listed twice in a set i.e., repeated elements being deleted.

For example, the set of letters in the word 'feed' is {f, e, d}. Though 'feed' has two 'e' s, only one 'e' appears in set notation.

REPRESENTATION OF A SET OR
METHOD OF DENOTING A SET OR
SPECIFICATION OF A SET

A set can be denoted or represented or specified symbolically by Three different Methods.

(A) Roster Method or Tabulation Method or Tabular Form or Enumeration Method

In this method, the elements of the Set are written in flowered brackets. This method is used when all the elements of a set are known and also few in numbers.

Examples :

1. A = {a, b, c, d}
2. B = {1, 2, 3, 4, 5}
3. C = { a, e, i, o, u}
4. D = {12, 15, 18, 21, 24}

(B) Rule Method or Set Builder Form

In this method also, the elements of a set are written in flowered brackets. This method is used when elements of a set have a specific property P and if any object satisfying the property, P is the element of the Set.

If x is any element (called representative element) of a set A having the property P, then A is denoted as

A = {x/x has the Property P}

Examples :

For example, let A be the set of all odd numbers. Then usually we use the letter 'x' to represent the representative element and we write,

a) A = {x/x is odd}

which is read as "A is the set of numbers x such that x is odd"

The vertical line '/' or ':' is read as "Such that".

b) A = {x : x is an odd number < 9}

i.e., A = {1, 3, 5, 7} – Tabular Form

c. $B = \{x/x \text{ is a vowel}\}$
i.e., $B = \{a, e, i, o, u\}$ – Tabular Form

d. $C = \{x : x \text{ is a multiple of } 3 : 10 < x < 25\}$
i.e., $C = \{12, 15, 18, 21, 24\}$ – Tabular Form

e. $D = \{x/x \text{ is a multiple of } 5\}$
i.e., $D = \{5, 10, 15....\}$ – Tabular Form

(C). Descriptive Phrase Method

A set can also be denoted by stating in words what its elements are :

Examples:

a. A is a set of first five natural numbers.
$A = \{x : x \text{ is a natural number less than } 6\}$ – Set Builder Form
$A = \{1, 2, 3, 4, 5\}$ – Tabular Form

b. The set of first 10 natural numbers.
$A = \{x : x \text{ is a natural number less than } 11\}$ – Set Builder Form
$A = \{x : x \in N, x \leq 10\}$ – Set Builder Form
$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ – Tabular Form

Examples:

I. Rewrite the following examples of the sets using Tabular Form and Set Builder Form:

1. The numbers 2, 4, 6, 8, 10 and 12
2. All even integers
3. The numbers 5, 7, 9, 11, 13 and 15
4. Set of letters p, q, r, s, and t.

Answers:

1. $\{2,4,6,8,10,12\}$ – Tabular Form
$\{x/x \text{ is an even natural number} \leq 12\}$ – Set Builder Form

2. $(\pm 2, \pm 4, \pm 6...\}$ – Tabular Form
$\{x/x \text{ is an even integer}\}$ – Set Builder Form

3. $\{5,7,9,11,13,15\}$ – Tabular Form
$\{x : x \text{ is an odd positive integer and } 5 \leq x \leq 15\}$ – Set Builder Form

4. $\{p, q, r, s, t\}$ – Tabular Form
$\{x/x \text{ is one of the five letters after o in English alphabets}\}$ – Set Builder Form

II. State in words

a. $A = \{x/x \text{ lives in Delhi}\}$
b. $B = \{x : x \text{ speaks Tamil}\}$
c. $C = \{x/x \text{ is older than 35 years}\}$
d. $D = \{x : x \text{ is an English living in India}\}$
e. $E = \{x/x^2 = 16\}$

8. C is the set of Indians who are non-vegetarian.
9. D is the set of people who love Hindi.
10. E is the set of people who always speak truth.
11. A = {3, 5, 7, 9, 11, 13, 15, 17, 19}
12. B = {17, 26, 35, 44, 53, 62, 71, 80}
13. C = {0, ± 2, ± 4, ± 6 ...}
14. D = {2, 4, 6, 8, 10 ...}
15. E = {2}

Answers:

1. {x : x is an odd number < 13}
2. {x/x = – 3n – 1, n = 0,1,2 ....}
3. {x:x is a vowel}
4. {x:x is a multiple of 5,4 < x < 26}
5. {x:x is a multiple of 3 }
6. A = {x/x is a book on set theory}
7. B = {x/x takes wine}
8. C = {x/x is an Indian and non –vegetarian}
9. D = {x/x loves Hindi}
10. E = {x/x always speaks truth}
11. A = {x/x is an odd number 1 < x < 21}
12. B = {x:x is an addition of 9 ; 17 ≤ x ≤ 80}
13. C = {x/x is an even integer}
14. D = {x/x is even and positive}
15. E = {x/x – 1 = 1} = {x/3x = 6}

FORMS OF SETS

A. Finite Sets

Sets which contain finite number of elements are called "Finite Sets".

Examples

a. Set of positive integers less than 10.
b. Set of odd numbers between 0 and 100
c. The students of a class
d. A = {a,e,i,o,u}
e. B = {1, 2, 3}

B. Infinite sets

Sets which contain infinite number of elements are called "Infinite Sets".

Examples:

a. Set of positive integers
b. Set of odd numbers
c. Set of points on a line
d. Set of even numbers
e. Set of negative integers

Examples:

Classify the following sets as Finite and Infinite:

1. Set of all rivers in India
2. Set of all capital cities in the world
3. Set of all professors in our college
4. Set of commodities rice, wheat, shoes
5. Set of all odd numbers
6. Set of all integers greater than 5 but less than 10 including 10
7. Set of letters a,b,c,d,e
8. Set of all straight lines in a plane
9. ϕ
10. {1, 2, 3, 4, 5, ...}
11. {1, 2, 3, 4, 5, }
12. {x/x is odd}

Answers:

Finite: 1, 2, 3, 4, 6, 7, 9, 11
Infinite: 5, 8,10, 12

C. Empty set or Null set or Void set or Zero set

A set which does not contain any element is called an Empty Set. It is also called as 'Null Set'or 'Void Set' or 'Zero Set'. The empty set is denoted by ϕ or { }. The empty set is a subset of every non-Null sets.

Examples:

1. A = {x/x is a letter before A in the English alphabet}
2. B = {x/x is an integer 8 < x < 9}
3. C = {x/x is a positive number less than – 2}
4. E = {x/x is a positive number less than 0}
5. F = {x/x is a man 500 years old}
6. G = {x/x is a boy aged 2 in IIB.A.}
7. H = {x/x is an odd integer and x is divisible by 2}
8. I = {x:x is pregnant male}

Examples:

Determine which of the following sets are null sets:

1. A = {x : x is a natural number and x < 1}
2. B = {x ∈ N, x is an even and x is prime}
3. C = {x/x + 16 = 16}
4. D = {x/x² + 1 = 0, x is real}

Answer:

Null Sets: 1 and 4

D. Subsets

A set 'A' is a 'Subset of a set B if and only if each element of set 'A' is also an element of the set B.

That is 'A' is a subset of 'B' if and only if

$x \in A$ implies

$x \in B$.

We denote this by $A \subseteq B$ which is read as "A is a subset of B". If A is not a subset of B, we denote this by $A \not\subseteq B$.

Properties of Subsets

1. Every set is a subset of itself, i.e., $A \subseteq A$ for any set A.
2. The Null set is a subset of every set, i.e., $\phi \subseteq A$ for any set A.
3. If A is a subset of B and B is a subset of C, then A is a subset of C, i.e., $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$, for any sets A, B and C.

Example:

Let us consider the sets A = {a, i} and B = {a, e, i, o, u}. We find that all the elements of the set A are also the elements of the set B. In such case, we say that "A is a subset of the set B".

E. Super Set

In the above example, A is the subset of the set B. Then we say that B is a 'Super Set' of A. We denote this by $B \supseteq A$..

F. Proper Subset

A set 'A' is a "Proper Subset" of a set B, if and only if each element of the set A is an element of the set B and at least one element of the set B is not an element of the set A.

We denote this by $A \subset B$ which is read as "A is a proper subset of B".

If A is not a proper subset of B, we denote this by $A \not\subset B$.

Example :

Let us consider the sets A = {2,3,4,5} and B = {3,2,4,5,6}. We find that all the elements of the set A are also the elements of the set B and one element of the set B (i.e., 6) is not an element of the set A. In such case, we say that "A is a proper subset of the set B".

G. Improper Subset

A set 'A' is called an "Improper Subset of set B", if and only if A = B. That is every set is "Improper Subset of itself".

Example:

1. Let A = {1,2,3,4} and B = {4,2,1,3}. Here A = B. Therefore, A is Improper Subset of B.

H. Universal Set or Master Set

When we are considering a set 'A', we assume the existence of another set 'X' or 'U' and form the set 'A' by selecting either all elements or some of the elements of 'X'. Such a set 'X' is called an "Universal Set" or "Master Set".

Universal set is denoted by 'U' or 'X'.

Examples:

1. While considering the set of all students of a college, the Universal Set consists of all students of the University.
2. If we consider the set of all electrified villages in India, the Universal Set consists of all villages in India.
3. While considering the set of all even integers, the Universal Set consists of all integers.

I. Unit Set or Singleton Set

A Set containing only one element is called "Unit set" or "Singleton Set".

Examples:

1. A = {a} is an Unit Set.
2. B = {0} is an Unit Set.
   The set B = {0} contains only one element i.e., 0, and hence it is an Unit Set but not a Null Set.
3. C = {1} is an Unit Set.

J. Equal Sets or Identical Sets or Equivalant Sets or Equality of Sets

Two sets A and B are 'Equal' or "Identical Sets" if and only if they contain the same elements. We denote this by

A = B.

This is known as "Axiom of Extension" or "Axiom of Identity".

Examples :

1. If A = {1, 2, 3, 4} and B = {2, 1, 4, 3} then A = B
2. If A = {Price, Demand, Income} and
   B = {Demand, Income, Price)
   then A = B.

K. Power Set of a Set

The collection or class or family of all subsets of a set A is called the "Power Set of A". We denote this power set of a set A by P (A).

P(A) = {x:x is a subset of A}

If 'n' is the number of elements in a set, then the total number of subsets will be 2^n. This is the reason why the collection of sets of A is called the power set of A.

Examples:

1. If A = {0,1} then

   P(A) = [ϕ, {0}, {1}, {0,1}]

2. If S = {a,b,c} then

   P(S) = [ϕ, {a}, {b}, {c}, {a,b}, {b,c}, {c,a}, {a,b,c}].

L. Comparability of Sets

Two sets A and B are said to be comparable if A is a subset of B or B is a subset of A, i.e.,

$A \subseteq B$ or $B \subseteq A$

The sets are said to be not comparable if A is not a subset of B and B is also not a subset of A i.e.,

$A \not\subseteq B$ or $B \not\subseteq A$.

Examples :

1. The sets A = {2,3} and B = {1, 2, 3,} are comparable since $A \subseteq B$.
2. The sets A = {a,e,i,o,u} and B = {o,u} are comparable since $B \subseteq A$.
3. The sets A = {1, 2, 5} and B = {2, 5, 6} are not comparable since,

   $1 \in A$ but $1 \notin B$ and

   $6 \in B$ but $6 \notin A$.

Some Particular Sets of Numbers bearing definite name and Notation

1. N – The set of all Natural Numbers
2. Q – The set of all Rational Numbers
3. Q_+ – The set of all Positive Rational Numbers
4. Q_o or Q^* – The set of Non–zero Rationals
5. R – The set of all Real Numbers
6. R_+ – The set of all Positive Real Numbers
7. R_o or R^* – The set of Non-zero Real Numbers
8. Z or I – The set of all Integers
9. Z_+ – The set of all Positive Int
10. Z_o or z^* – The set of non–zero Intege

11. Z_- – The set of all Negative Integers
12. E – The set of Even Integers.
13. C – The set of all Complex Numbers
14. C_0 or C^* – The set of non–zero Complex Numbers
15. P – The set of all Prime Numbers.

SET OPERATIONS OR ALGEBRA OF SETS

In this topic, we define the operations of Union, Intersection, Disjoint, Difference and Complement of Sets.

(A) Union of Sets or Set Union or Sum

Union or sum of two sets A and B is the set of all elements of both A and B. We denote this by $A \cup B$ and is read as "A Union B". In symbols,

$A \cup B = \{x : x \in A, \ x \in B\}$

Examples :

1. If $A = \{1,2,3,\}$ and $B = \{3,4,5\}$, then
   $A \cup B = \{1,2,3,4,5\}$.

2. If $A = \{3,5,6\}$ and $B = \{1,3,6,9\}$, then
   $A \cup B = \{1,3,5,6,9\}$.

3. If $A = \{a,b,c,d\}$ and $B = \{f,b,d,g\}$, then
   $A \cup B = \{a,b,c,d,f,g\}$.

(B) Intersection of Sets

The intersection of two sets A and B is the set of the elements which are common to A and B. We denote this by $A \cap B$ and is read as "A Intersection B".

Hence $A \cap B = \{x : x \in A \text{ and } x \in B\}$. In some books $A \cap B$ is denoted by AB. The Intersection of A and B (i.e., $A \cap B$) is also called the "Product of A and B".

Examples :

1. If $A = \{1, 2, 3,\}$ and $B = \{3, 4, 5\}$, then
   $A \cap B = \{3\}$.

2. If $A = \{2, 3, 4, 5\}$ and $B = \{4, 5, 6, 7, 8\}$, then
   $A \cap B = \{4, 5\}$.

3. If $A = \{a, e, i, o, u\}$ and $B = \{k, l, o, u, j\}$ then
   $A \cap B \{o, u\}$.

4. If $A = \{8, 7, 9, 10\}$ and $B = \{10, 7, 11, 12\}$ then
   $A \cap B = \{7, 10\}$.

(C) Disjoint Sets

If two sets A and B have no common elements, then we say that A and B are "Disjoint Sets"

Hence, if A and B are Disjoint Sets, $A \cap B = \phi$.

Examples :

1. If $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$, then, A and B are disjoint sets.
2. If $A = \{ l,m,n,o\}$ and $B = \{ a,e,c,d\}$, then, A and B are disjoint sets.

(D) Difference of Sets or Set Differences

The Set Difference or the difference of two sets A and B is the set of all elements which belong to A but do not belong to B. we denote this by A-B and is read as " A difference B" or 'A minus B'.

Hence $A - B = \{ x : x \in A, x \notin B\}$

Examples:

1. If $A = \{ 1, 2, 3, 4,\}$ and $B = \{ 1,3, 4, 5, 6\}$, then
   $A - B = \{2\}$;
   $B - A = \{ 5,6\}$.
2. If $A = \{ a, b, c, d, e\}$ and $B = \{ a, c, e, g, h, i\}$, then
   $A - B = \{ b, d\}$;
   $B - A = \{ g, h, i\}$.

(E) Complement of A Set

Let 'X' be the Universal set and 'A' be the subset of the Universal set 'X'. Then the Complement of the set A is that of all elements of X which do not belong to A. We denote the Complement of set A by A' (A - Prime) or A_C.

Hence, $A' = \{ x/x \in X \text{ and } x \notin A\}$

i.e., $A' = X - A$.

Examples:

1. Let $U = \{ 1, 2, 3, 4, 5, 6\}$ and $A = \{ 2, 5, 6\}$, then
   $A' = \{ 1, 3, 4\}$.
2. Let X be set of all English Alphabets and A be the set of all Consonants.
   Therefore, $A' = X - A = \{ a, e, i, o, u\}$.
3. Let X be the set of all integers between 1 and 50 including 1 and 50; and let A be the set of all even integers between 1 and 50. Therefore A will contain all odd numbers between 1 and 50.
4. Let $U = \{ x / x \in N : 0 < x < 50\}$
   $A = \{ x/x \in N ; 0 < x < 40\}$
   then, $A' = \{ x / x \in N : 40 < x < 50$

Properties or Laws or Theorems of Set Theory

If A, B, and C are subsets of the Universal Set 'X' then the following laws hold:

(A) Commutative Laws

a. $A \cup B = B \cup A$ .... For Union

b. $A \cap B = B \cap A$ .... For Intersection

(B) Associative Laws

a. $A \cup (B \cup C) = (A \cup B) \cup C$ ..... For Union

b. $A \cap (B \cap C) = (A \cap B) \cap C$ ..... For Intersection

(C) Distributive Laws

a. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

b. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(D) Difference Laws

a. $A - B = A \cap B'$

b. $A - B = A - (A \cap B) = (A \cup B) - B$

(E) Complementary Laws

a. $A \cup A' = X ; A \cap A' = \phi$

b. $A \cup X = X \ (\because A \subset X) ; A \cap X = A$

c. $A \cup \phi = A; A \cap \phi = \phi$

(F) Demorgan's Laws of Complementation

a. $(A \cup B)' = A' \cap B'$

i.e., the Complement of the Union is equal to Intersection of the Complements.

b. $(A \cap B)' = A' \cup B'$

i.e., the Complement of the Intersection is equal to the Union of Com[illegible]

c. $A - (B \cup C) = (A - B) \cap (A - C)$

d. $A - (B \cap C) = (A - B) \cup (A - C)$

(G) Idempotent Laws

a. $A \cup A = A$

b. $A \cap A = A$.

VENN DIAGRAMS

Most of the relationships between the sets can be represented by diagrams. These diagrams are known as ' Venn Diagrams' or " Venn - Euler Diagrams". A rectangle is used to represent the " Universal Set 'U' or 'X'. All the elements of the Universal set will be represented by points in it. A circle inside the rectangle will represent a set which is obtained from 'U'. To every subset of 'U', we can associate a circle. Such a diagram is called a "Venn Diagram".

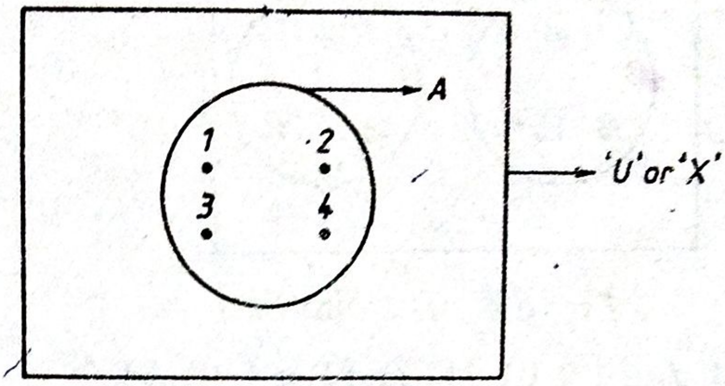


Fig. 8.1 : Universal Set and Sub Set

In the Figure 8.1, Set A = { 1,2, 3, 4} is a Subset of the Universal Set 'X' or 'U'.

Union of Sets

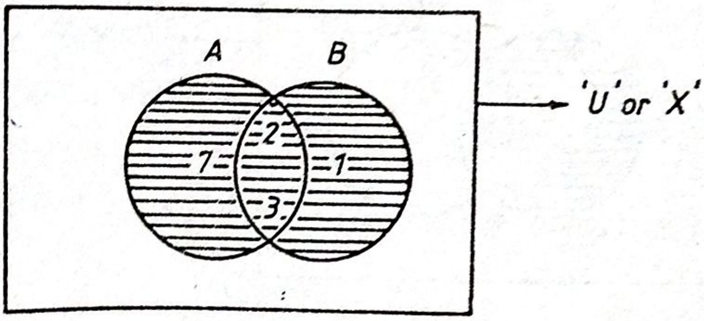


Fig. 8.2 : Union of Two Sets

In the Figure 8.2, A = { 2, 3, 7} and B = { 1, 2, 3}

Therefore, A ∪ B = { 1, 2, 3, 7}. This is represented by the shaded area in the Figure 8.2.

Intersection of Sets

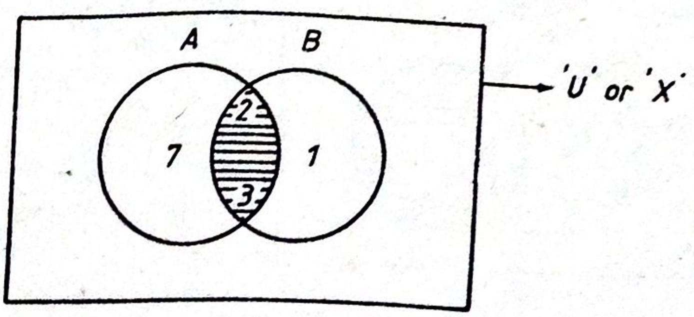


Fig. 8.3 : Intersection of Two Sets

In the Figure 8.3 A = { 2, 3, 7} and B = { 1, 2, 3}.

Therefore, $A \cap B$ = { 2, 3}. The shaded area in the Figure 8.3 represents the Intersection.

Disjoint Sets

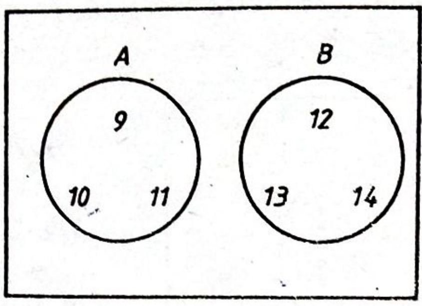


Fig. 8.4 : Disjoint Sets

In the Figure 8.4, A = { 9,10, 11} and B = { 12, 13, 14}. Therefore, A and B are "Disjoint Sets". (Since there is no common elements in the sets A and B).

Difference of Sets

(A). A – B

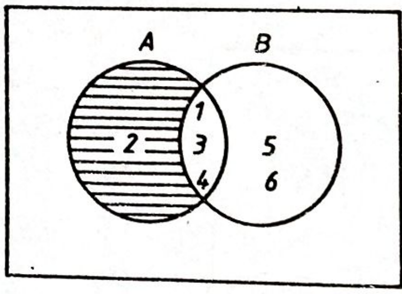


Fig. 8.5 : A–B–Difference of Two Sets A and B

In the Figure 8.5, A = {1, 2, 3, 4} and B = { 1, 3, 4, 5, 6}. Therefore, A–B = {2}. This is represented by the shaded area in the Figure 8.5.

(B) B – A

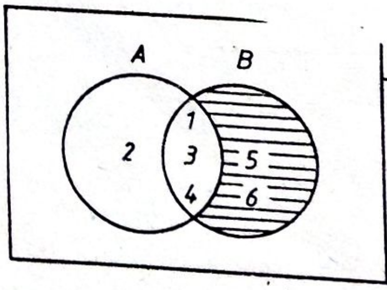


Fig. 8.6 : B–A–Difference of Two Sets B and A

In the Figure 8.6, A = { 1, 2, 3, 4} and B = { 1, 3, 4, 5, 6}.
Therefore, B – A = { 5, 6}, that is, the shaded area in the Figure 8.6 represents the situation.

Complement of the Set

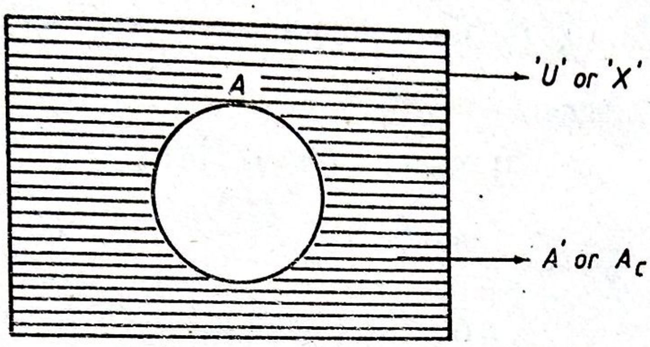


Fig. 8.7 : Complement of a Set

In the Figure 8.7, the shaded area represents the Complement of the set A.

ORDERED PAIRS

An ordered pair consists of two elements say 'a' and 'b'. One of these two, say 'a' is designated as the first element and the other 'b' as the second element. An ordered pair is represented by (a,b).

If (a,b) and (c,d) are two ordered pairs then,

(a,b) = (c,d), if and only if a = c and b = d.

Thus, the ordered pairs (3,5) ≠ (5,3) since 3 ≠ 5 and 5 ≠ 3. If (x, y) = (3,5), then x must be equal to 3 and y must be equal to 5.

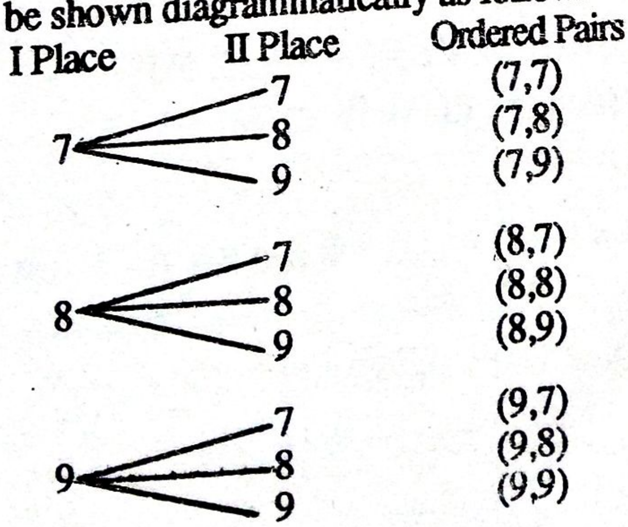
Examples:

1. Given a set S = { 7,8,9}, how many ordered pairs can be manufactured from S?

Solution:

There are two places in an ordered pair. We have three choices in the first place, because we have three elements.
Similarly, we have three choices in the second place. Thus there is a total of $3 \times 3 = 9$ ordered pairs.

This can be shown diagrammatically as follows:



(2) If we have two sets, A = { 3,4,5} and B = {6,7} how many ordered pairs exist?

Solution:

There are 3 choices for the first place and two choices for the second place. Therefore, there are $3 \times 2 = 6$ ordered pairs.

This can be shown diagrammatically as follows:

I Place	II Place	Ordered Pairs
3	6	(3,6)
	7	(3,7)
4	6	(4,6)
	7	(4,7)
5	6	(5,6)
	7	(5,7)

In general if the set 'A' has 'm' elements and the set 'B' has 'n' elements, we can have $m \times n$ ordered pairs.

CARTESIAN PRODUCT

Let A and B be any two non - empty sets. Then the Cartesian Product of these two non - empty sets A and B, is the set of all possible ordered pairs (a,b), where $a \in A$ and $b \in B$. We denote this Cartesian Product by $A \times B$ (read as A cross B). In symbols, $A \times B = \{ (a, b) / a \in A \text{ and } b \in B\}$

Thus the Cartesian Product of two non - empty sets is the set of the possible ordered pairs of the elements of two sets such that in each ordered pair, the first element belongs to the first set and the second element belongs to the second set.

Examples:

1. If A = { 1,2,3} and B = {4,5}, find i) $A \times B$ and ii) $B \times A$.

Solution:

i) $A \times B = \{ (1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

ii) $B \times A = \{ (4,1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3)\}$

Therefore $A \times B \neq B \times A$.

2. If A = { 1,2,3} and B = { 1,2} find $A \times B$ and $B \times A$.

Solution:

$A \times B = \{ (1,1), (1,2), (2,1), (2,2), (3,1), (3,2)\}$

$B \times A = \{ (1,1), (1,2), (1,3), (2,1), (2,2), (2,3)\}$.

3. Find $A \times B$ and $B \times A$ where $A = \{1,2,3\}$ and $B = \{a,b,c\}$.

Solution:

$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c), (3,a) (3,b), (3,c)\}$

$B \times A = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3), (c,1), (c,2), (c,3)\}$.

4. Find $A \times B$ and $B \times A$, if $A = \{1,2,3\}$ and $B = \{2,3,4\}$.

Solution:

$A \times B = \{(1,2), (1,3), (1,4), (2,2), (2,3) (2,4), (3,2), (3,3), (3,4)\}$

$B \times A = \{(2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (4,1), (4,2), (4,3)\}$.

5. If the set A contains 'm' elements and the set B contains 'n' elements, show that $A \times B$ contains $m \times n$ elements.

Solution:

$A = \{m\}$

$B = \{n\}$

$A \times B = \{(m,n)\}$

Therefore, $A \times B$ contains $m \times n$ elements.

Examples:

1. Let $A = \{x,y,z,u,v,w,p,q,r\}$

$B = \{u,v,w,a,b,c\}$ and

$C = \{l,m,n,o, p, q, r, x,y\}$

Let X be the Universal Set of all English alphabets. Verify the following relations for these sets :

a) $A \cup B = B \cup A$

b) $(A \cup B) \cup C = A \cup (B \cup C)$

c) $A \cap B = B \cap A$

d) $(A \cap B) \cap C = A \cap (B \cap C)$

e) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

f) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$.

Solution:

a) $A \cup B = B \cup A$

$A \cup B = \{a,b,c, p, q, r, u, v, w, x, y, z,\}$

$B \cup A = \{a, b, c, p, q, r, u,v, w, x, y, z\}$

Therefore, $A \cup B = B \cup A$.

b) $(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cup B) = \{a, b, c, p, q, r, u, v, w, x, y, z\}$

$(A \cup B) \cup C = \{a, b, c, l, m, n, o, p, q, r, u, v, w, x, y, z\}$

$(B \cup C) = \{a, b, c, l, m, n, o, p, q, r, u, v, w, x, y\}$

$A \cup (B \cup C) = \{a, b, c, l, m, n, o, p, q, r, u, v, w, x, y, z\}$

Therefore, $(A \cup B) \cup C = A \cup (B \cup C)$.

c) $A \cap B = B \cap A$

$A \cap B = \{u, v, w\}$

$B \cap A = \{u, v, w\}$

Therefore, $A \cap B = B \cap A$.

d) $(A \cap B) \cap C = A \cap (B \cap C)$

$A \cap B = \{u, v, w\}$

$(A \cap B) \cap C = \{\}$ or ϕ

$B \cap C = \{\}$ or ϕ

$(B \cap C) \cap A = \{\}$ or ϕ

Therefore, $(A \cap B) \cap C = (B \cap C) \cap A$.

e) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

$A \cup B = \{a, b, c, p, q, r, u, v, w, x, y, z\}$

$(A \cup B) \cap C = \{p, q, r, x, y\}$

$A \cap C = \{p, q, r, x, y\}$

$B \cap C = \{\}$

$(A \cap C) \cup (B \cap C) = \{p, q, r, x, y\}$

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$.

f) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

$A \cap B = \{u, v, w\}$

$(A \cap B) \cup C = \{l, m, n, o, p, q, r, u, v, w, x, y\}$

$A \cup C = \{l, m, n, o, p, q, r, u, v, w, x, y, z\}$

$B \cup C = \{a, b, c, l, m, n, o, p, q, r, u, v, w, x, y\}$

$(A \cup C) \cap (B \cup C) = \{l, m, n, o, p, q, r, u, v, w, x, y\}$

Therefore, $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$.

2. If $A = \{1, 2, 3\}$, $B = \{1, 3, 4, 5\}$ and $C = \{2, 4, 5, 6\}$, verify the identify.

a) $(A \cup B) \cup C = A \cup (B \cup C)$

b) $(A \cap B) \cap C = A \cap (B \cap C)$

c) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and

d) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Solution:

a) $(A \cup B) \cup C = A \cup (B \cup C)$

$A \cup B = \{1, 2, 3, 4, 5\}$

$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6\}$

$B \cup C = \{1, 2, 3, 4, 5, 6\}$

$A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6\}$

Therefore, $(A \cup B) \cup C = A \cup (B \cup C)$.

b) $(A \cap B) \cap C = A \cap (B \cap C)$

$A \cap B = \{1,3\}$

$(A \cap B) \cap C = \{\}$

$B \cap C = \{4,5\}$

$A \cap (B \cap C) = \{\}$

Therefore, $(A \cap B) \cap C = A \cap (B \cap C)$.

c) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$B \cup C = \{1, 2, 3, 4, 5, 6\}$

$A \cap (B \cup C) = \{1, 2, 3\}$

$A \cap B = \{1,3\}$

$A \cap C = \{2\}$

$(A \cap B) \cup (A \cap C) = \{1,2,3\}$

Therefore, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

d) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$(B \cap C) = \{4, 5\}$

$A \cup (B \cap C) = \{1, 2, 3, 4, 5\}$

$(A \cup B) = \{1, 2, 3, 4, 5\}$

$(A \cup C) = \{1, 2, 3, 4, 5, 6\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5\}$

Therefore, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

3. Let $A = \{a, b, c, d, e, f\}$ $B = \{c, d, e\}$ and $C = \{a, e\}$. Let U be the Universal set of all English Alphabeticals, verify

(a) $(A \cup B)' = A' \cap B'$ (b) $(A \cap B)' = A' \cup B'$

(c) $(B \cup C)' = B' \cap C'$ (d) $A - (B \cup C) = (A - B) \cap (A - C)$.

Solution:

(a) $(A \cup B)' = A' \cap B'$

$A \cup B = \{a,b,c,d,e,f\}$

$(A \cup B)' = \{g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

$A' = \{g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

$B' = \{a, b, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

$A' \cap B' = \{g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

So, $(A \cup B)' = A' \cap B'$.

(b) $(A \cap B)' = A' \cup B'$

$(A \cap B) = \{c, d, e\}$

$(A \cap B)' = \{a, b, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, v, w, x, y, z\}$

$A' \cup B' = \{a, b, f, g, h, i, j, l, m, n, o, p, q, r, s, t, v, w, x, y, z\}$

So, $(A \cap B)' = A' \cup B'$.

(c) $(B \cup C)' = B' \cap C'$

$(B \cup C) = \{a, c, d, e\}$

$(B \cup C)' = \{b, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

$C' = \{b, c, d, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

$B' \cap C' = \{b, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

So, $(B \cup C)' = B' \cap C'$.

(d). $A - (B \cup C) = (A - B) \cap (A - C)$

$A - (B \cup C) = \{b, f\}$

$A - B = \{a, b, f\}$

$A - C = \{b, c, d, f\}$

$(A - B) \cap (A - C) = \{b, f\}$

So, $A - (B \cup C) = (A - B) \cap (A - C)$.

4. Let $A = \{1, 2, 3, 4\}$

$B = \{2, 4, 6, 8\}$ and

$C = \{3, 4, 5, 6\}$,

verify that

(a) $A \cup B = B \cup A$

(b) $A \cap B = B \cap A$.

(c) $(A \cup B) \cup C = A \cup (B \cup C)$

(d) $(A \cap B) \cap C = A \cap (B \cap C)$.

Solution :

(a) $A \cup B = B \cup A$

$A \cup B = \{1, 2, 3, 4, 6, 8\}$

$B \cup A = \{1, 2, 3, 4, 6, 8\}$

So, $A \cup B = B \cup A$.

(b) $A \cap B = B \cap A$

$A \cap B = \{2, 4\}$

$B \cap A = \{2, 4\}$

So, $A \cap B = B \cap A$.

(c) $(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cup B) = \{1, 2, 3, 4, 6, 8\}$

$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 8\}$

$(B \cup C) = \{2, 3, 4, 5, 6, 8\}$

$A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6, 8)$

So, $(A \cup B) \cup C = A \cup (B \cup C)$.

(d) $(A \cap B) \cap C = A \cap (B \cap C)$

$(A \cap B) = \{2, 4\}$

$(A \cap B) \cap C = \{4\}$

$(B \cap C) = \{4, 6\}$

$A \cap (B \cap C) = \{4\}$

So, $(A \cap B) \cap C = A \cap (B \cap C)$.

EXERCISE 8.1

(A) Rewrite the following statements using Set Notation:

1. X does not belong to A.
2. B is a Super Set of A.
3. D is a member of E
4. F is not a Sub Set of G.
5. H does not include D.

(B) State in words and then write in Tabular Form :

1. $A = \{x / x^2 = 4\}$.
2. $B = \{ x: x - 2 = 5\}$.
3. C = {x / x is positive, x is negative}.
4. D = { x/x is a letter in the word 'Pandian'}.

(C) Write the following sets in Set Builder Form :

1. Let A consist of the letters a, b, c, d and e.
2. Let B = {2, 4, 6, 8 ...}
3. Let C consist of the Countries in the United Nations
4. Let D = {3}
5. Let E be the Prime Ministers Indira Gandhi, Rajiv Gandhi, Narasimha Rao.

Here 3, 2, 1 are the Coefficients of x, y and z respectively.

FUNCTIONS AND THEIR GRAPHICAL REPRESENTATION

Before attempting to know the meaning of a ' Function', it is imperative to know the meaning of ' Relation'. The meaning of relation can be explained by an illustration.

For example, Demand Curve (Straight Line) $D = 25 - 3P$ constitutes a "Relation" between D and P, where D is the Demand and P is the price. If we have the situation where there is only one D value for each P, then the Relation is called a " Function". That is, in the above example "D is the Function of P".

Therefore, relationship between the variables is called a "Function". In other words, a "Function" is a special case of a " Relation".

A "Function" is called "Mapping" or "Transformation" — both words connote the action of associating one thing with another.

Example:

Let x and y be two variables and if the value of the variable 'y' depends upon the value of the variable 'x', we say that 'y' is a 'Function' of 'x' and we can write it symbolically as,

$y = f(x)$.

Examples in Economics:

1. Demand Function

If 'P' is the price and 'x' is the quantity of a commodity demanded by the consumers, we write the "Demand Function" as,

i) $P = f(x)$ or $x = f(P)$

ii) $P = f(q)$ or $q = f(P)$

where, 'q' is the quantity demanded. It conveys that quantity demanded depends upon price. The slope of the Demand Curve is 'Negative' in general.

2. Supply Function

a) If 'P' is the price and 'x' is the quantity of a commodity supplied by the firm, we write the "Supply Function" as,

i) $P = f(x)$ or $x = f(P)$

ii) $P = f(q)$ or $q = f(P)$

Where 'q' is the quantity supplied. It conveys that supply depends upon price.

b) If 'U' refers to various components of input and 'x' is the quantity of a commodity supplied by the firm, we write the 'Supply Function ' as,

i) $U = f(x)$ or $x = f(U)$

ii) $U = f(q)$ or $q = f(U)$

where 'q' is the quantity of a commodity supplied by the firm. It conveys that supply depends upon inputs.

3. Savings Function

If 'S' stands for savings and 'y' stands for income, we write the "Savings Function" as,

$$S = f(y)$$

It conveys that savings depend upon income.

4. Production Function

If 'x' is the amount of output produced by the firm, when the factors or inputs Land (D), Labour (L), Capital (K) and Organisation (O) are employed, we write the "Production Function" as,

$$x = f(D, L, K, O)$$

or

$$q = f(D, L, K, O)$$

where 'q' is the amount produced by the firm. It conveys that production depends upon four factors of production.

5. Cost Function

If 'x' is the quantity produced by a firm at a total cost 'C', we write the "Cost Function" as,

$$C = f(x)$$

It conveys that cost depends upon the quantity produced.

6. Revenue Function

If 'R' is the Total Revenue of a firm, 'Q' is the quantity demanded or sold and 'P' is the price per unit of output, we write the "Revenue Function" as,

$R = f(Q,P)$

or

$R = f(Q)$ ... {since $P = f(Q)$}

It conveys that revenue depends upon quantity sold.

7. Utility Function

If 'U' is the Utility of a consumer and q_1 and q_2 are the quantities of commodities consumed, we write the "Utility Function" as,

$$U = f(q_1, q_2)$$

It conveys that utility depends upon the quantities of commodities consumed.

8. Profit Function

If 'x' is the quantity produced by a firm, 'R' is the total revenue and 'C' is the total cost, we write the " Profit Function" as,

$$\pi = R(x) - C(x)$$

where, π = profit.

9. Forms or Types or Classifications of Functions

There are many types of functions. But, let us now discuss the functions which are found in economics.

1. Linear or First Degree Function

A function f(x) is " Linear" or "First Degree" if 'x' (independent variable) occurs in it only in the first degree. In other words, a Polynomial Function of degree one is a " Linear Function".

Examples :

1. $y = mx$ is a Linear Function, where m is a real number.
2. $y = mx + c$ is a First Degree Function, where 'm' is slope and 'c' is y–intercept.
3. $y = 2x + 3$ is a First Degree Function

Then $\frac{dy}{dx} = 2$

The value of $\frac{dy}{dx}$ (i.e., Co - efficient of x) is the 'slope' or 'gradient' of the equation. In the example, $\frac{dy}{dx} = 2$. Therefore, the slope is 2. The slope or gradient of the equation may be positive(+) or negative (–). If the slope is negative, then the line is a decreasing straight line and if the slope is positive, then the line is an increasing straight line.

i) If x = 0, then the value of y is the "Y - intercept" or " Intercept on Y axis". Hence,

$$\begin{aligned} y = 2x + 3 &= 2(0) + 3 \\ &= 0 + 3 \\ &= 3 \end{aligned}$$

Therefore, "Y - intercept" or "Intercept on Y axis" is equal to 3.

ii) If y = 0, then the value of x is called the "X–intercept" or "Intercept on X –axis". Hence,

$$y = 2x + 3$$
$$0 = 2x + 3$$
$$2x + 3 = 0$$
$$\therefore\ 2x = -3$$
$$x = -3/2$$

Therefore, the "X–intercept" or "Intercept on X axis" is equal to 3/2. The Table 2.2 for x and y is obtained from the above equation.

Table 2.2

x	0	–3/2
y	3	0

The graph of the function can be plotted from the Table 2.2.

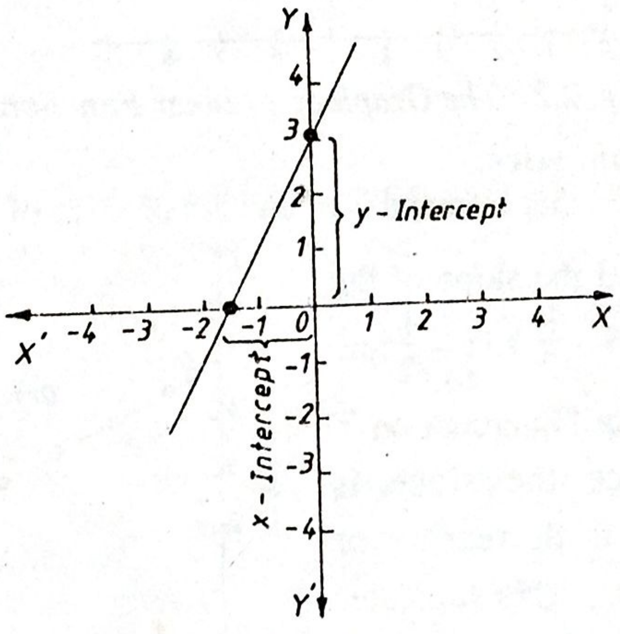


Fig. 2.1 : Intercepts

The "Y - intercept" denotes the point at which the straight line cuts the vertical axis (Y axis) and is represented by the value of y, when x is equal to 0.

The "X - intercept" denotes the point at which the straight line cuts the horizontal axis (X axis) and is represented by the value of x, when y is equal to 0.

The intercept may be positive (+) or negative (–).

Graphical Representation

The graph of the Linear Function is a "Straight Line" which is not parallel to the co - ordinate axes.

Let us find the graphical representation of the Linear equation y = 2x + 3. The Table 2.3 for x and y is obtained from the above equation.

Table 2.3

x	0	1	2	3	4	5
y	3	5	7	9	11	13

The graph of the function can be plotted from the Table 2.3.

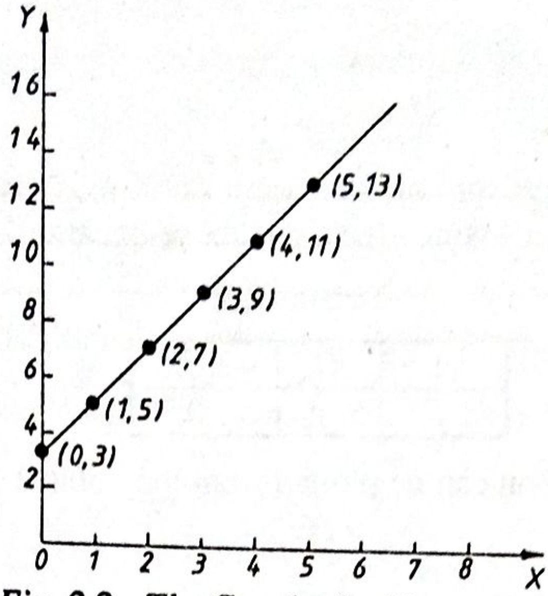


Fig. 2.2 : The Graph of a Linear Function

Example in Economics:

Marshall assumed that Demand is a function of price of commodity.

In the Figure 2.3 the slope of the function (i.e., $D = 5 - \frac{P}{2}$) is $-\frac{1}{2}$ and the "Y - intercept" or "Intercept on Y axis" is 5. Since the slope is negative, the line is decreasing or sloping downwards. This indicates that the Demand Curve slopes downwards.

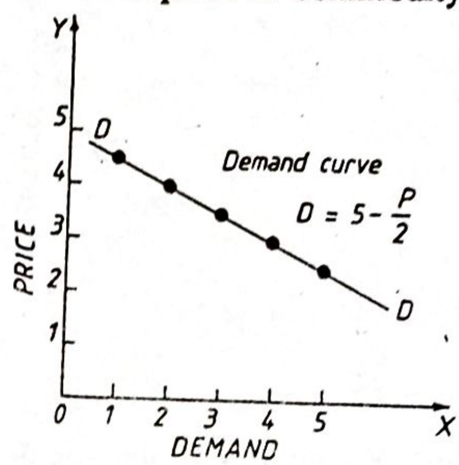


Fig. 2.3 : Demand Line

2. Quadratic Function or Second Degree Function

This is a Non - Linear Function with maximum power 'Two' (2) either in 'x' or 'y' or both in x and y. In other words, a Polynomial Function of degree two is a "Quadratic Function".

Examples:

1. $y = ax^2 + bx + c$

   where a, b and c are real numbers and $a \neq 0$.
2. $y^2 = 4ax$
3. $y = 2x^2 - 4x + 1$ ... Quadratic Function in a single variable

4. $x^2 + y^2 = 15$
5. $Q = P^2$
6. $ax^2 + by^2 + cxy + d$ ... Quadratic Function in two variables.

Graphical Representation

The graph of a "Quadratic Function" is a "Parabola" i.e., "U shaped". The same example $y = 2x^2 - 4x + 1$ can be taken for graphical representation.

The Table 2.4 for x and y is obtained from the above equation.

Table 2.4

x	−1	0	1	2	3
y	7	1	−1	1	7

The graph of the function can be plotted from the Table 2.4

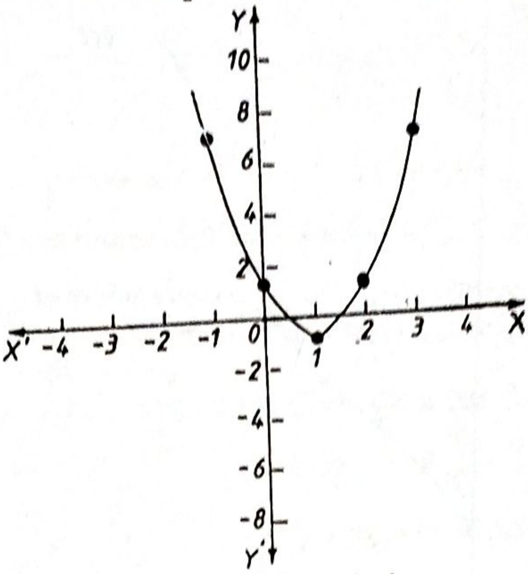


Fig. 2.4 : A Parabola

Example in Economics:

Marginal Cost and Average Cost are Second Degree Functions of the level of output.

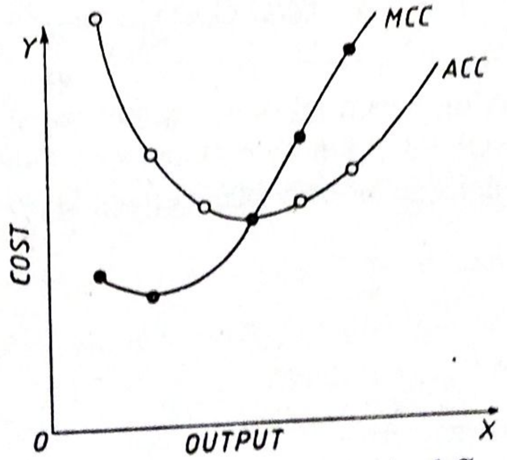


Fig. 2.5 : Average Cost and Marginal Cost Curves

3. Cubic or Third Degree Function

This is a Non - Linear Function with maximum power *"Three" (3)* either in "x" or "y" or both in "x" and "y". In other words, a Polynomial Function of degree three is a *"Cubic Function"*.

Examples:

1. $Y = ax^3 + bx^2 + cx + d$
   where a, b, c and d are real numbers.
2. $Y = 3x^3 + 2x^2 + x + 5$ ... cubic function in a single variable.
3. $Z = ax^3 + bx^2y + cy^3 + dxy^2$ ... cubic function in two variables.

Graphical Representation (Example in Economics)

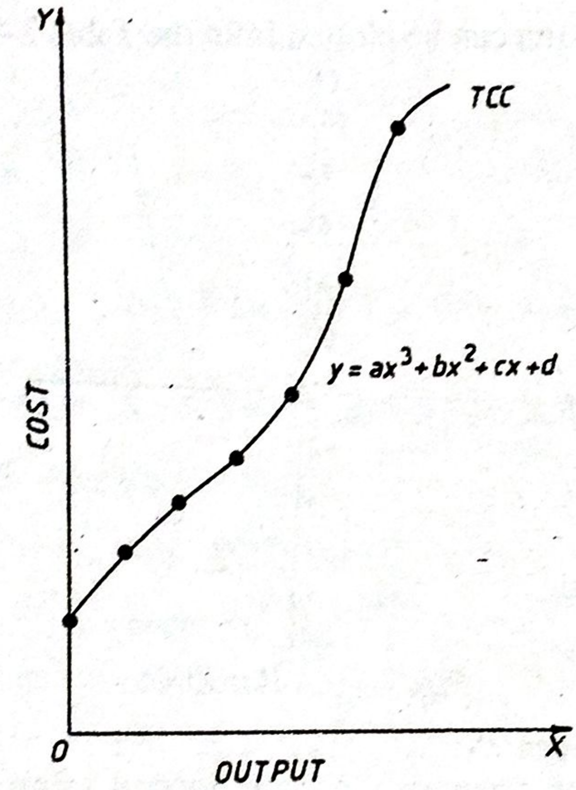


Fig. 2.6 : Total Cost Curve

4. Constant Function

It is a type of function which takes a constant value. In other words, a zero degree Polynomial Function is a "Constant Function". That is, a function whose range consists of only one element is said to be a *"Constant Function"*.

Examples:

1. $Y = a$
2. $X = b$ where, a and b are constants.

Graphical Representation

The graph of the constant function is a *"Straight Line Parallel to X axis or Y axis"*.

(a) y is constant

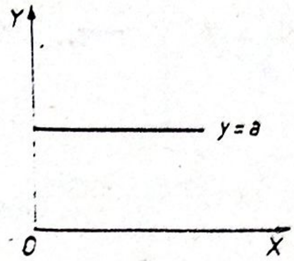


Fig. 2.7 : Straight Line Parallel to X axis

(b) x is constant

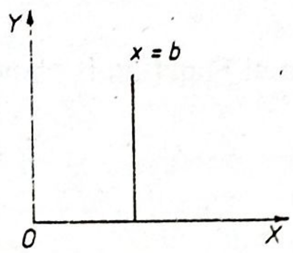


Fig. 2.8 : Straight Line Parallel to Y axis

Example in Economics:

(c) In Economics the Total Factor Cost (TFC) is a constant function of level of output.

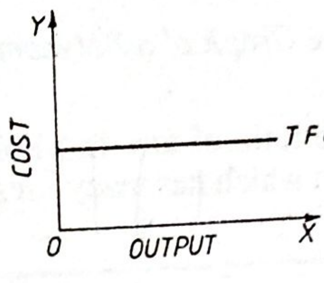


Fig. 2.9 : Total Factor Cost Curve

5. Polynomial Function:

Polynomial means "Multi-term" and the "Polynomial Function" is a general function in x and y of the form.

$$Y = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_n$$

Where $a_0, a_1, a_2 \ldots\ldots a_n$ are real numbers and 'n' is a non - negative integer.

Evidently, Polynomial Functions are generalisations of the Linear, Quadratic and Cubic Functions. Actually, the situation is reverse. The Linear, Quadratic and Cubic Functions are particular cases of Polynomial Functions.

Therefore, $x = 3$ is a root of the equation $3x - 9 = 0$, because the value of x (i.e., 3) satisfies the given equation.

A) LINEAR EQUATIONS

1. Linear Equation is $ax + b = 0$.
   The solution to this equation is $ax = -b$

   Therefore, $x = -\frac{b}{a}$

2. $$\begin{aligned} 3x + 5 &= 20 \\ 3x &= 20 - 5 \\ 3x &= 15 \\ x &= \frac{15}{3} = 5. \end{aligned}$$

3. $$\begin{aligned} 3x - 9 &= 0 \\ 3x &= 9 \\ x &= \frac{9}{3} = 3 \end{aligned}$$

4. $$\begin{aligned} 9x + 9 &= x + 12 \\ 9x - x &= 12 - 9 \\ 8x &= 3 \\ x &= \frac{3}{8}. \end{aligned}$$

5. $$\begin{aligned} 3(x + 5) &= 21 \\ 3x + 15 &= 21 \\ 3x &= 21 - 15 \\ 3x &= 6 \\ x &= \frac{6}{3} \\ x &= 2 \end{aligned}$$

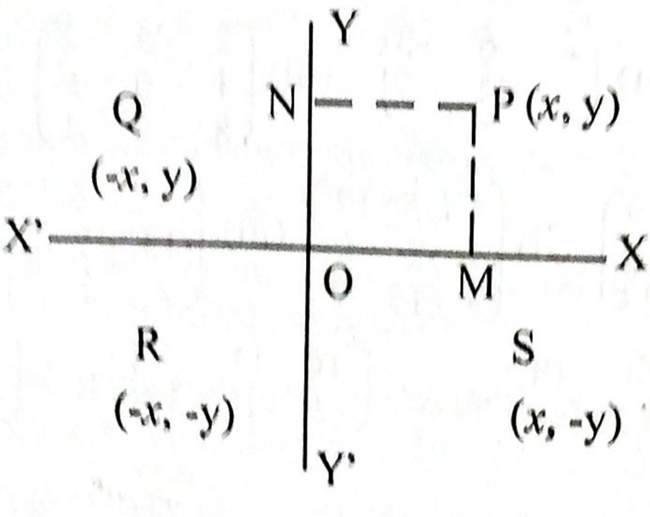
ANALYTICAL GEOMETRY

Analytical Geometry was invented by the French Mathematician Ren'e Descartes (1596-1650). Analytical geometry is a study of geometry through algebra. Some problems of geometry can easily be solved with the help of algebraic techniques. These problems can be translated into algebra in order to solve it and again it can be translated into geometry.

CO-ORDINATES

The element in geometry is point and the element of algebra is number. The point in a plane may be represented in number.

In order to represent a point in number draw two strainght lines intersecting with each other at right angles (90^0). Intersection of the two straight lines is shown as follows.



The two straight lines are intersecting at a point denoted as O. The point is called 'origin'. The line XOX' is called X-axis and the line YOY' is called Y-axis. These two axis are also known as reference lines or co-ordinates. The two co-ordinates divide the plane into four parts called quadrants.

X co-ordinate is also called 'abscissa'. The area right of Y axis represents positive of X co-ordinate and the area left of Y axis represents negative of X co-ordinate. Y co-ordinate is also called 'ordinate'.

The area above the X axis is positive of Y co-ordinate and the area below Y axis is the negative of Y co-ordinate.

In the diagram point P means, (x,y); point Q means $(-x,y)$; point R means $(-x,-y)$ and point S means $(x,-y)$. Each point measures value corresponding to the two axis. For example if straight lines are drawn from point P to X axis and Y axis, the line cut X axis at M and the line cut Y axis at N. The distance of OM is the numerical value of x and the distance of ON is the numerical value of y representing the point P.

Distance Between Two Points

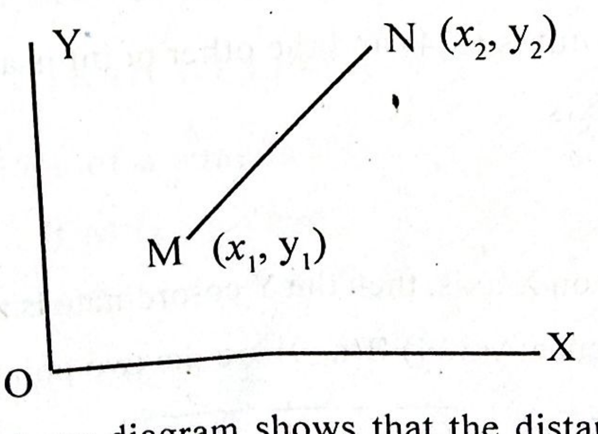
If M (x_1, y_1) and N (x_2, y_2) are two points in a plane, then the distance between these two points is measured by the following formula.

$$MN = \sqrt{(x_2-x_1)^2 + (y_2 - y_1)^2}$$

The distance from origin to any point in the plane is calculated as

$$OM = \sqrt{x_1^2 + y_1^2}$$

The following diagram shows the representation of two points in a plane.



The above diagram shows that the distance between the two points M and N.

Example: 1 Calculate the distance between the following points

(i) (3, 7), (4, 8)
(ii) (0, 0), (2, 4)
(iii) (-4, -3), (-3, 5)

Solution :

Let the points in each case be P and Q.

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(i) $PQ = \sqrt{(4 - 3)^2 + (8 - 7)^2}$

$= \sqrt{1^2 + 1^2} = \sqrt{2}$

(ii) $PQ = \sqrt{(2 - 0)^2 + (4 - 0)^2}$

$= \sqrt{2^2 + 4^2} = \sqrt{4 + 16}$

$= \sqrt{20}$

(iii) $PQ = \sqrt{(-3-(-4))^2 + (5 - (-3))^2}$

$= \sqrt{(-3+4)^2 + (5+3)^2}$

$= \sqrt{1^2 + 8^2} = \sqrt{1 + 64}$

$= \sqrt{65}$

Example: 2

If the distance between two points is $\sqrt{17}$ such that the co-ordinates of one point is (3,4) and the other point is at X axis, find the point on the X axis.

Solution :

If the point is on X axis, then the Y co-ordinate is zero. Assume the point as (a, 0). Hence A(3, 4) B(a, 0) are the two points.

Distance of a line $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{17} = \sqrt{(a-3)^2 + (0 - 4)^2}$

$17 = (a-3)^2 + 16$

$17 = a^2 - 6a + 9 + 16$

$a^2 - 6a + 25 - 17 = 0$

$a^2 - 6a + 8 = 0; \quad a^2 - 4a - 2a + 8 = 0$

$a(a-4) - 2(a-4) = 0; \quad (a - 4)(a - 2) = 0$

$\therefore a = 2 \text{ or } 4.$

Example: 3

If A(3, 7) B(-2, x) and C(5, 2) are co-ordinates of three points and AB = AC, find the value of x.

Solution:

$$\text{Distance of a line} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Given AB} = \text{AC}$$

$$(-2-3)^2 + (x-7)^2 = (5-3)^2 + (2-7)^2$$

$$25 + (x-7)^2 = 4 + 25$$

$$25 + (x-7)^2 = 29$$

$$(x-7)^2 = 29 - 25 = 4$$

$$x-7 = \pm 2$$

$$x-7 = 2 \quad \text{or } x - 7 = -2$$

$$x = 9 \quad (\text{or}) \quad x = 5$$

SLOPE OF A STRAIGHT LINE

The slope of a straight line is also called as 'Gradient of a Straight Line'. If M (x_1, y_1) and N (x_2, y_2) are any two points in a straight line, then the slope of the line is measured as,

$$\text{Slope of MN} = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } \frac{y_1 - y_2}{x_1 - x_2}$$

The slope of a straight line is defined as the ratio between the vertical separation and horizontal separation of the two points. The vertical and horizontal separation of the two points are shown in the following diagram.

$a(a-4) - 2(a-4) = 0; \quad (a-4)(a-2) = 0$

$\therefore a = 2$ or 4.

Example: 3

If A(3, 7) B(-2, x) and C(5, 2) are co-ordinates of three points and AB = AC, find the value of x.

Solution:

Distance of a line = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Given AB = AC

$(-2-3)^2 + (x-7)^2 = (5-3)^2 + (2-7)^2$

$25 + (x-7)^2 = 4 + 25$

$25 + (x-7)^2 = 29$

$(x-7)^2 = 29 - 25 = 4$

$x-7 = \pm 2$

$x-7 = 2$ or $x - 7 = -2$

$x = 9$ (or) $x = 5$

SLOPE OF A STRAIGHT LINE

The slope of a straight line is also called as 'Gradient of a Straight Line'. If M (x_1, y_1) and N (x_2, y_2) are any two points in a straight line, then the slope of the line is measured as,

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The slope of a straight line is defined as the ratio between the vertical separation and horizontal separation of the two points. The vertical and horizontal separation of the two points are shown in the following diagram.

SIMPLE INTEREST, COMPOUND INTEREST AND ANNUITIES

I. SIMPLE INTEREST

Interest is the extra money paid for the use of anothers money. (Eg) X borrowed Rs. 5,000 from Y and X repaid the money at the end of the 2nd year. X has to pay an extra money for the usage of Y's money for 2 years. The extra money is called the simple interest . If the extra money (ie., the interest) is 10 % per annum, then X has to pay (5000 x10/100 x2) Rs.1000/-as interest.

The formula for calculating the Simple Interest is

SI = Pni

A = P+SI or P +Pni or P(1+ni)

P = SI/ni

n = SI/Pi

i = SI/Pn

Here P is the Principal; SI is the Simple Interest; A is the Amount (P+SI); n = number of years; i = rate of interest.

ILLUSTRATED PROBLEMS

1. Calculate Simple Interest and Amount for the following investments.
   - (i) Rs.10,000 for 3 years @ 15% per annum
   - (ii) Rs. 15,000 for $2\frac{1}{2}$ years @ 12 % per annum
   - (iii) Rs. 20,000 for 40 months @ 12 % per annum
   - (iv) Rs.25,000 for 190 days @ 10% per annum
   - (v) Rs.30,000 for 2 years 3 months and 20 days @ 15% per annum.

Solution :

SI = Pni, A = P+SI

SI = 10,000 x 3 x 15/100 = Rs.4500

A = 10,000 + 4500 = Rs.14,500

(ii) $SI = 15{,}000 \times 2.5 \times 12/100 \quad = \text{Rs. } 4{,}500$

$A = 15{,}000 + 4{,}500 \quad = \text{Rs. } 19{,}500$

(iii) $SI = 20{,}000 \times (40/12) \times (12/100) \quad = \text{Rs. } 8{,}000$

$A = 20{,}000 + 8{,}000 \quad = 28{,}000$

(iv) $SI = 25{,}000 \times (190/365) \times (10/100) = \text{Rs. } 1{,}301.37$

$A = 25{,}000 + 1{,}301.37 \quad = \text{Rs. } 26{,}301.37$

(v) $SI = 30.000 \times (2 + 3/12 + 20/365) \times 15/100$

$$= 30{,}000 \times \frac{8{,}760 + 1{,}095 + 240}{4380} \times \frac{15}{100}$$

$= 4{,}500 \times (10{,}095/4{,}380) \quad = \text{Rs. } 10{,}371.58$

$A = 30{,}000 + 10{,}371.58 \quad = \text{Rs. } 40{,}371.58$

2. Find the number of years in which an investment of Rs. 30,000 @ 12% Simple Interest p.a amounted to Rs. 39,000

Solution

$A = \text{Rs. } 39{,}000 \; ; \; P = \text{Rs. } 30{,}000$

$SI = A\text{-}P = 39{,}000 - 30{,}000 = \text{Rs. } 9000$

$i = Pni \, , \; SI = Pni$

$$n = \frac{SI}{Pi} = \frac{9000}{30{,}000 \times 12/100} \quad \frac{9 \times 100}{30 \times 12} = 2.5$$

i.e., 2 years and 6 months.

3. Find out the date on which an investment of Rs. 20,000 become Rs. 24,800 @ 8% Simple Interest p.a., if the date of investment is 1st January 2000.

Solution :

$P = \text{Rs. } 20{,}000; \; A = \text{Rs. } 24{,}800; \; i = 8\% = 0.08$

$SI = A\text{-}P = 24{,}800 - 20{,}000 = \text{Rs.} 4800$

$SI = Pni \; ; \; SI = Pni$

$n = SI/Pi = 4{,}800/ (20{,}000 \times 0.08) \; = 3 \text{ years}$

The date is 1st January 2003.

4. Find out the date on which an amount of Rs. 40,000 invested on 1st February 2001 become Rs. 42,300 @ 12% simple interest p.a

Solution :

P = Rs. 40,000 A - Rs. 42,300 i = 12% = 0.12

SI = A -P = 42,300 - 40,000 = Rs,2,300

SI = Pni

n = SI/Pi = 2,300/ (40,000 x 0.12) = 0.479 years

or 0.48 x 12 = 5.76 months

or 0.48 x 365 = 175 days.

The date is, (Feb.28 + March 31 +April 30 + May 31 +June 30

= 150; 175 - 150 = 25 in July)

= July 26, 2001.

5. X invested Rs.25,000 on 1st Jan 1998 for a period of $2\frac{1}{2}$ years, which gave him a simple interest of Rs. 6,250. Find out the rate of interest

Solution :

P = Rs 25,000; SI = Rs.6,250; n = $2\frac{1}{2}$ years

SI = Pni

i = SI /Pn = 6,250 / (25,000 x 2.5) = 10 /100 = 10%

6. An investment of Rs 20,000 becomes Rs 20,800 after 73days. Find the rate of Simple Interest

Solution :

A = Rs.20,800 P = 20,000

SI = A - P = 20,800 - 20,000 = Rs.800

N = 73days i.e 73/365 years or 1/5 years

SI = Pni

$$i = \frac{Si}{Pn} = \frac{800}{20,000 \times 1/5} = \frac{800 \times 5}{20,000} = \frac{20}{100} = 20\%$$

7. A has two sons. He left Rs 1,50,000 for his two sons. According to his will, the amount should be invested in a bank @ 12% simple interest p.a., so that, the two sons should get the same amount when they attained the age of 20. The elder son was 17 year old and the younger son was 14 years old when the man died. Find out the amount of investment for the two sons.

Solution :

Let the investment for the elder son be = x;

$i = 12\% = 0.12 \qquad n = 20 - 17 = 3$ years

$A_1 = P(1 + ni) = x(1 + 3 \times 0.12) = x(1 + 0.36) = 1.36x$

The investment for the younger son P = 1,50,000 - x

$i = 12\% = 0.12; \qquad n = 20 - 14 = 6$ years

$A_2 = P(1+ni) = 1.50{,}000 - x(1 + 6 \times 0.12)$

$= 1{,}50{,}000 - x(1+0.72) = (1{,}50{,}000 - x)\, 1.72$

$= 2{,}58{,}000 - 1.72x$

Since $A_1 = A_2$

$1.36x = 2{,}58{,}000 - 1.72x$

$1.36x + 1.72x = 2{,}58{,}000$

$3.08x = 2{,}58{,}000$

$x = 2{,}58{,}000 / 3.08 =$ Rs.83,766

Investment for the elder son = Rs.83,766

Investment for the younger son = 1,50,000 - 83,766

= Rs.66,234.

8.Calculate the amount invested when it gives a simple interest Rs. 15,000 for a period of 2 1/2 years @ 12% interest per annum.

Solution :

SI = Rs.15,000; $n = 2\frac{1}{2}$ years; $i = 12\% = 0.12$

SI = Pni

$$P = \frac{SI}{ni} = \frac{15{,}000}{2.5 \times 0.12} = \frac{15{,}000}{0.3} = \text{Rs.}50{,}000$$

9. Calculate the principal for a simple interest of Rs.12,000 @ 12% interest p.a for a period of 4 years 3 months and 17 days.

Solution :

SI = Rs. 12,000; n = 4 years 3 months and 17 days

$= 4 + 3/12 + 17/365 = 4.3$ years; $i = 12\% = 0.12$

SI = Pni

$$P = \frac{SI}{ni} = \frac{12{,}000}{4.3 \times 0.12} = \frac{12{,}000}{0.516} = \text{Rs.}23{,}256.$$

Solution :

Amount at the end of 5 th year = Rs. 51,200

Amount at the end of 3 rd year = Rs. 43,520

Simple Interest for 2 years = 51,200 - 43,520 = Rs.7,680

Simple Interest for one year = 7,680/2 = Rs.3840

Simple Interest for 3 years = 3,840 x 3 = Rs.11,520

$$A = P + SI$$

$$43{,}520 = P + 11{,}520$$

$$P = 43{,}520 - 11{,}520 = Rs.\ 32{,}000$$

$$\text{Rate of simple Interest} = \frac{3{,}840 \times 100}{32{,}000} = 12\%$$

II Compound Interest

Compound interest is the interest calculated for the principal and the interest for the prefixed period. Hence,at the time of calculation of interest the accumulated amount (i.e. Principal and the accumulated interest) is taken into account

Compound Interest is Calculated by the following formula

$$C.I = P(1+i)^n - P$$

$$A = P(1+i)^n$$

$$P = \frac{A}{(1+i)^n}$$

If interest compounded half yearly, then

$$C.I = P(1+i/2)^{2n} - P$$

If interest compounded quartely, then

$$C.I = P(1+i/4)^{4n} - P$$

If interest compounded monthly, then

$$C.I = P(1+i/12)^{12n} - P$$

Where C.I = Compound Interest; i = rate of Compound Interest; n = number of years; A = Total amount = P + C.I; P = Principal

ILLUSTRATED PROBLEMS

1. Calculate Compound Interest and the Amount for the following investment

Solution :

$P = Rs.14,000;$ $A = Rs,20,849;$ $i = 12\% = 0.12$

$A = P(1+i)^n$

$20,849 = 14,000\ (1+0.12)^n$

$20,849 = 14,000 \times 1.12^n$

$1.12^n = 20,849 / 14,000 = 1.4892$

$\log 1.12^n = \log 1.4892$

$n \log 1.12 = \log 1.4892$

$n \times 0.04922 = 0.172953$

$\therefore n = 0.172953 / 0.04922 = 3.5$ years

6. **Find the rate of compound interest for an investment of Rs.12,000 which became Rs.23,105 at the end of 5th year.**

Solution :

$P = Rs.12,000;$ $A = Rs.23,105;$ $n = 5$

$A = P(1+i)^n$

$23,105 = 12,000\ (1+i)^5$

$(1+i)^5 = 23,105 / 12,000 = 1.9254$

$\log (1+i)^5 = \log 1.9254$

$5 \log (1+i) = \log 1.9254 = 0.284521$

$\log (1+i) = 0.284521/5 = 0.0569$

$(1+i) = \text{Antilog } 0.0569$

$1 + i = 1.14$

$i = 1.14 - 1 = 0.14 = 14\%$

7. **An amount of investment became Rs.28,092 at the end of the second year and Rs.35,870 at the end of the fourth year. Calculate the investment and the rate of compound interest.**

Solution :

Let A_1, and A_2 be the amount at the end of 2 nd and 4 th years respectively.

$A = P(1+i)^n$

$A_1 = P(1+i)^2 = 28,092$

$A_2 = p(1+i)^4 = 35,870$

...y with equal intervals. payment

METHOD OF CALCULATING ANNUITY

1. CALCULATION OF IMMEDIATE ANNUITY

Immediate Annuity means the payment of fixed amount at the ... of the period. The accumulated amount at the end of the specified period ... calculated as follows.

$$A = P/i\,[(1+i)^n - 1]$$

Where A = Amount; P = Annuity; i = rate of compound interest; n = number of the periods.

Example :1 X Deposits Rs.7000 in a bank at the end of each year ... 12% compound interest for a period of 9 years. Find ... total amount at the end of the 9th year if the intalments ... allowed to accumulate.

Solution :

P = 7000; i = 12% = 0.12; n = 9

$$A = \frac{P}{i}\left[(1+i)^n - 1\right] = \frac{7000}{0.12}\,[(1+0.12)^9 - 1]$$

$= 58{,}333\,(1.12^9 - 1)$ $= 58{,}333\,(2.773 - 1)$

$= 58{,}333 \times 1.773$ = Rs.103,424

[$\log 1.12^9 = 9 \log 1.12 = 9 \times 0.04922 = 0.44298$;

Antilog 0.44298 = 2.773]

Example :2 A Person deposits in a bank at the end of each month Rs.40... for a period of 2 years @ 10% compound interest.

Find total amount in the credit of his account at the end of the 2 nd year.

Solution

$P = 4,000 \qquad i = 10\%/12 = 0.1/12 = 0.0083$

$n = 2$ years ie; $2 \times 12 = 24$ instalments

$$A = \frac{P}{i}[((1+i)^n - 1)] = \frac{4,000}{0.0083}[(1+0.0083)^{24} - 1]$$

$= 4,81,928 (1.0083^{24} - 1) = 4,81,928 (1.2196 - 1)$

$= 4,81,928 \times 0.2196 = $ Rs. 1,05,831

[$\log (1.0083)^{24} = 24 \log 1.0083 = 24 \times 0.00359 = 0.0862$;
Antilog 0.0862 = 1.2196]

2. CALCULATION OF AMOUNT OF ANNUITY DUE

Annuity Due is the payment of equal instalments in thebeginning of the period. The accumulated amount of Annuity Due is calculated as follows.

$$A = P\left(\frac{1+i}{i}\right)[(1+i)^n - 1]$$

Where A = Amount; P = Annuity; i = rate of compound interest and n = number of instalments.

Example : X invests Rs. 5000 every year in a financial institution in the beginning of a year for 12 years. If the financial company pays a compound interest @ 15% p.a., find the accumulated amount.

Solution :

$P = $ Rs.5,000; $n = 12$; $i = 15\% = 0.15$

$$A = P\left(\frac{1+i}{i}\right)[(1+i)^n - 1]$$

$= 5000 (1+0.15/0.15) [(1+0.15)^{12} - 1]$

$= 5000 \times 7.67 \times (1.15^{12} - 1)$

$= 38,350 (5.35 - 1)$

$= 38,350 \times 4.35 = $ Rs. 1, 66,823

[$\log 1.15^{12} = 12 \log 1.15 = 12 \times 0.060698 = 0.72838$;
Antilog 0.72838 = 5.35]

3. CALCULATION OF PRESENT VALUE OF ANNUITY

The present value of accumulated annuity is calculated as follows.

$$V = \frac{P}{i}\left[1-(1+i)^{-n}\right] \quad \text{or} \quad = \frac{P}{i}\left[1-\frac{1}{(1+i)^{n}}\right]$$

Where, V = Present value; P = Annuity;
i = Rate of compound interest; n = Number of instalments.

Example :1 Calculate the present value for an yearly investment of Rs. 6,000 @ 10% compound interest for a period of 7 years.

Solution :

$P = 6000;$ $i = 10\% = 0.10;$ $n = 7$

$$V = \frac{P}{i}\left[1-(1+i)^{-n}\right] = \frac{6,000}{0.1}\left[1-(1+0.1)^{-7}\right]$$

$= 60,000\,[1-(1.1)^{-7}] \quad = 60,000\,[1-(1/1.1^{7})]$

$= 60,000\,[1-(1/1.9487)] = 60,000 \times (1-0.5132]$

$= 60,000 \times 0.4868 \quad = \text{Rs.}29,208$

[$\log 1.1^{7} = 7 \log 1.1 = 7 \times 0.041393 = 0.28975$

Antilog $0.28975 = 1.9487$]

Example :2 X bought a motorcycle by paying Rs.10,000 as down payment and the balance amount in 10 equal annual instalments of Rs. 3,000 each. The instalments include compound interest@ 9% p.a If X buys the vehicle by cash payment, find out how much he has to pay.

Solution:

Instalment Price = Rs.10,000 + (3000 x 10) = Rs.40,000

Cash Price = Rs. 10,000 + Present value of Rs.30,000

$P = 3,000$ $i = 9\% = 0.09;$ $n = 10$

$$\text{Present Value } V = \frac{P}{i}\left[1-(1+i)^{-n}\right] = \frac{3,000}{0.09}\left[1-(1+0.09)^{-10}\right]$$

$= 33333.33\ 1-(1.09)^{10} \quad = 33333.33\,(1-{}^{1}/_{1.09}{}^{10})$

$= 33333.33\,[1-(1/2.3626)] = 33333.33 \times [1-0.4233]$

$= 33333.33 \times 0.5767 = \text{Rs. } 19,223$

Cash Price $= 10,000 + 19,223 = \text{Rs. } 29,223$

[$\log 1.09^{10} = 10 \log 1.09 = 10 \times 0.03743 = 0.3743$

Antilog $0.3743 = 2.3676$]

CHAPTER IX

RATIO AND PROPORTION

I. RATIO

Ratio means the relation between two quantities of the same kind. It explains to what extent one quantity is greater or smaller than the other quantity.

If X and Y are the two quantities, then the ratio between X and Y is X/Y. It may be represented as X:Y. If both the quantities are of same value then the ratio will be equal to one. (Eg.) When the marks scored by A is 90 and B is 30, than the ratio of marks scored by A and B is 90:30 i.e, 90/30 = 3. It means the score of marks by A is 3 times greater than the score of marks by B.

The first quantity of a ratio is called 'antecedent' and the second quantity is called 'consequent'. The antecedent is represented in the numerator and the consequent is represented in the denominator.

(Eg) The ratio between the daily income of A and B is 100:150 i.e, 100/150. (or 2:3 i.e, 2/3). In this ratio 100 is called antecedent which is represented in the numerator and 150 is called the consequent, which is represented in the denominator.

Two quantities of different kind could not be expressed in ratio. (Eg.) The marks scored by one individual and the income of another individual could not be expressed in ratio.

TYPES OF RATIOS

1. Inverse Ratio

If k:g is a ratio then g:k is the inverse of k:g. The product of these two ratios is one. (Eg.) If 2:3 is a ratio then 3:2 is inverse of the ratio 2:3. i.e., 2:3 = 2/3; 3:2 = 3/2 , Product of these two ratios = 2/3 x 3/2 = 1

2. Compound Ratio

Compound ratio is the product of two or more ratios. To obtain a compound ratio the antecedes of one ratio with antecedes of another ratio; and the consequent of one ratio with the consequent of another ratio should be multiplied.

(Eg.) If $x:y$ and $k:g$ are the two ratios, representing related characteristic then the compound ratio is $xk : yg$.

Compound ratio may be in the form of 'duplicate ratio' as $x^2 : y^2$; 'triplicate ratio' as $x^3 : y^3$; 'sub-duplicate ratio' as $\sqrt{x} : \sqrt{y}$; or 'sub-triplicate ratio' as $\sqrt[3]{x} : \sqrt[3]{y}$.

3. Continued Ratio

Continued Ratio means the relation between 3 or more quantities of the same kind. (Eg.) k : g : n is a compound ratio of three quantities. k , g and n of same kind; k : l : m : n is a compound ratio of four quantities k, l, m and n of same kind.

Commensurable and Incommensurable Quantities

The quantities expressed in a ratio with integers is called commensurable quantities. (Eg.) 3 : 5 The quantities expressed in a ratio with real numbers (except integers) is called incommensurable quantities. (Eg) 1.5 : 2.7 or $\sqrt{3} : \sqrt{5}$ or $\sqrt[3]{4} : \sqrt[5]{3^2}$ etc.

Illustrated Problems

1. Find the ratio of

(i) 25 to 75
(ii) 30cm to 2m
(iii) Rs. 7 to 75 paise
(iv) 1 hour to 22 minutes

Solution:

(i) 25 to 75 is 25 : 75 or 1 :3 i.e; 1/3

(ii) 1 metre = 100 cms
2 meters = 200 cms
Hence, 30 cm to 2m is 30 : 200 or 3:20, i.e., 3/20.

(iii) Rs. 7 = 700 paise. Hence. Rs. 7 to 75 paise is 700 : 75 or 28 : 3 i.e., 28/3

(iv) 1 hour = 60 minutes
Hence, 1 hour to 22 minutes is 60:22 or 30:11 i.e., 30/11

TYPES OF PROPORTION

1. Direct Proportion:

If increase or decrease of one ratio results in increase or decrease of another ratio, then these two ratios are said to be directly proportional. (Eg.) a : b is directly proportional to c : d if increase or decrease of a : b is directly proportional to the increase or decrease of c :d

2. Inverse Proportion:

If an increase in one ratio results in the decrease of another ratio and vice versa, these two ratios are said to be inversely proportional. (Eg.) a :b is inversaly proportional to c ; d, if increase of a : b results in the decrease of c : d.

3. Simple Proportion and Compound Proportion:

Simple proportion is the method of calculating the fourth proportional from the three given quantities. It is also known as Rule of three.

Compound proportion is the repeated use of the simple proportion. It is also known as Double Rule of three.

4. Continued Proportion:

If a : b = b : c, then the quantities a, b, c are said to be, in continued proportion. Here 'b' is called the 'mean proportional' between a and c. c is called the third proportional to a and b.

ILLUSTRATED PROBLEMS

1. Find the fourth proportional to 2, 5, 6

Solution:

Let the fourth proportional be x

Let the quantities are 2, 5, 6, x

Hence, $2 : 5 = 6 : x$

i.e., $2/5 = 6/x$; $2x = 6 \times 5 = 30$

$x = 30 / 2 = 15$

Hence, the fourth proportional to 2, 5, 6 is 15

2. Find the third proportional to 10 and 20

Solution:

Let x be the third proportional

Hence, $10 : 20 = 20 : x$; i.e., $10 / 20 = 20 / x$

$10x = 20 \times 20 = 400$; $x = 400 / 10 = 40$

3. Find the mean proportional between 4 and 16

Solution:

Let x be the mean proportional

Hence, $4 : x = x : 16$; i.e., $4 / x = x / 16$

$x \cdot x = 4 \times 16$; $x^2 = 64$; $x = \sqrt{64} = 8$

4. If $(x + 4) : (9 - 2x) = 6 : 5$ find the value of x.

Solution:

$(x + 4) : (9 - 2x) = 6 : 5$

$\frac{x + 4}{9 - 2x} = \frac{6}{5}$; $6(9 - 2x) = 5(x + 4)$

$54 - 12x = 5x + 20$; $12x + 5x = 54 - 20$

$17x = 34$; $\therefore x = 34 / 17 = 2$

5. If a : b = c : d = 4 : 9, find the value of ad : bc and a + b : c + d

Solution:

a : b = 4 : 9; i.e. a / b = 4 / 9; ∴ a = 4; b = 9; c : d = 4 : 9

∴ c = 4; d = 9

ad : bc = 4 x 9 : 9 x 4 = 36 : 36 = 1 : 1

a + b : c + d = 4 + 9 : 4 + 9 = 13 : 13 = 1 : 1

6. If a + b : a – b = 11 : 6, find the ratio b : a

Solution:

a + b : a – b = 11 : 6

$\frac{a + b}{a - b} = \frac{11}{6}$; $6(a + b) = 11(a - b)$