MANONMANIAM SUNDARANAR UNIVERSITY

DIRECTORATE OF DISTANCE & CONTINUING EDUCATION

TIRUNELVELI 627012, TAMIL NADU

B.Sc. STATISTICS - III YEAR

DJS3C - STATISTICAL QUALITY CONTROL

(From the academic year 2016-17)

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DJS3C - STATISTICAL QUALITY CONTROL

Unit - I

Quality control and need for statistical quality control techniques in industries - causes of variation - process control and product control. Specifications and tolerance limits-3σ limits, construction of Shewhart control charts - variable control charts - $\overline{X}$, R and σ charts.

Unit - II

Control charts for attributes: control chart for fraction defectives (p chart), number of defectives (d chart) and number of defects per unit (c chart).

Unit - III

Acceptance Sampling - Sampling inspection, producer's risk and consumer's risk-acceptable quality level (AQL), lot tolerance percent defective (LTPD), average outgoing quality level (AOQL), ATI and ASN. Rectifying inspection plans.

Unit - IV

Acceptance sampling by attributes: Single sampling plan - OC, AOQ, ATI and ASN curves - Double sampling plan and its advantages over single sampling plan, Operating procedure.

Unit - V

Acceptance sampling for variables-sampling plan based on normal distribution-known and unknown standard deviation cases. Determination of n and k for one- sided specification limits - OC curve.

BOOKS FOR STUDY:

1. Montgomery, D.C. (1991) Statistical Quality Control (2nd Edition) John Wiley and Sons, New York.
2. Eugene L. Grant, and Richard S. Leavenworth (1988) Statistical Quality Control (Sixth Edition), McGrawhill Book co, New York.
3. Gupta, S. C. and V.K. Kapoor (1999) Fundamentals of Applied Statistics (Third Edition), Sultan Chand & sons, New Delhi.
4. Goon, A. M., M.K. Gupta and B. Dasgupta (1987) Fundamentals of Statistics, Vol. II. World Press, Kolkata.
5. Mahajan (1997) Statistical Quality Control, Dhanpat Rai & sons, New Delhi.
6. Juran, J.M.(1988) Quality Control Handbook, McGraw Hill, New York.

Unit I

Basics and Control Charts

1.1. Introduction

1.2. Basics in Statistical Quality Control

1.3. Control Charts

1.4.Variable Control Charts

1.1. Introduction

Every manufacturing organisation is concerned with the quality of its product. While it is important that quality requirements be satisfied and production schedules met, it is equally important that the finished product meet established specifications. Because, customer's satisfaction is derived from quality products and services. Stuff competition in the national and international level and consumer's awareness require production of quality goods and services for survival and growth of the company. Quality and productivity are more likely to bring prosperity into the country and improve quality of work life.

However, the management looks to achieve customer satisfaction by running its business at the desired economic level. Both these can be attained by properly integrating quality development, quality maintenance and quality improvement of the product. The integration of these three aspects of a product can be achieved through a sound quality control system.

The Meaning of "Quality"

Quality is a relative term and it is generally used with reference to the end of the product. For example, a gear used in sugarcane juice extracting machine may not possess good surface finish, tolerance and accuracy as compared with the gear used in the head stock of a lathe, still it may be considered of good quality if it works satisfactorily in the juice extracting machine. The quality is thus defined as the fitness for use/purpose at the most economical level.

The quality depends on the perception of a person in a given situation. The situation can be user-oriented, cost-oriented or supplier-oriented. Since, the item is manufactured for the use of the customer, the requirements of the customer dictates the quality of the product. Quality is to be planned, achieved, controlled and improved continuously.

The word "Quality" has variety of meanings:

1. **Fitness for purpose**

The component is said to possess good quality, if it works well in the equipment for which it is meant. Quality is thus defined as fitness for purpose.

2. **Conformance to requirements**

Quality is the ability of the material/component to perform satisfactorily in an application for which it is intended by the user. Quality of a product, thus, means conformance to requirements. Customer needs have to be assessed and translated into specifications depending upon the characteristics required for specific application. Just as every human has his own characteristics every application has its own characteristics.

An example, let us consider a fountain pen. The application of the fountain pen is to write on the paper in order to perform this function satisfactorily, the required characteristics are:

- It should hold sufficient quantity of ink, so that frequent refilling is avoided.
- It should regulate the flow of ink into the nib.
- It should mark the characters on the paper. The marking should be neither too thin nor too broad.
- It should not tear the paper.
- It should be of convenient size to hold between fingers.
- It should have a good appearance.
- It should prevent ink from drying when not in use.
- It should not be slippery nor should it hurt fingers.
- It should hold securely to the pocket.
- It should not be too expensive.
- It should have a reasonable life. It should sustain reasonable shocks (unbreakable).

Depending on these demands, it is necessary to decide the length, diameter, material, tip of the nib etc. Hence, the demands of the application are translated into the requirements and the requirements are quantified. These quantified requirements are called specifications.

3. **Grade**

Quality is a distinguishing feature or grade of the product in appearance, performance, life, reliability, taste, odour, maintainability etc. This is generally called as quality characteristic.

4. **Degree of Preference**

Quality is the degree to which a specified product is preferred over comparing products of equivalent grade, based on comparative test by customer's, normally called as customer's performance.

5. **Degree of Excellence**

Quality is a measure of degree of general excellence of the product.

6. **Measure of Fulfilment of Promises**

The quality of a product is a measure of fulfilment of the promises made to the customers.

7. In terms of product characteristics, Feigenbaum defines quality as:

"The total composite product and service characteristics of engineering, manufacturing, marketing and maintenance through which the product and service in use meet the expectation of the customers".

The key point of this definition is that quality depends mainly on customer's perception as described earlier. Hence, it is essential that all these features must be built in the design and maintained in manufacturing which the customer would like to have and is willing to pay for it.

For example, the product must perform its intended function repeatedly a called upon, over its stipulated life cycle under normal conditions of use. It is required that the product must look attractive and be safe in handling. It should last for a longer period and be economical. It should be easy to operate or use.

Thus we conclude that the product should have certain abilities to perform satisfactorily in a stated application. These abilities may be categorised ten factors as under:

1. **Suitability:** For specific application.
2. **Reliability:** It should give efficient and consistent performance.
3. **Durability:** It should have desired life.
4. **Workability:** Safe and fool proof workability.
5. **Affordability:** It should be economical.
6. **Maintainability:** It should be easy to maintain.
7. **Aesthetic look:** It should look attractive.
8. **Satisfaction to Customers:** It should satisfy the customer's requirements.
9. **Economical:** It should have reasonable price.
10. **Versatility:** It should serve number of purposes.

A product can be said to possess good quality if all the above requirements are properly balanced while designing and manufacturing it.

Quality Control

Control

Control can be defined as a process by means of which we observe performance and compare it with some standard. If there is a deviation between the observed performance and the standard performance then it is necessary to take corrective action.

Quality Control

The term quality control has variety of meanings:

1. Quality control is the process through which we measure the actual quality performance, compare it with the standards and take corrective action if there is a deviation.
2. It is a systematic control of variation factors that affect the quality of the product. It depends on: Material, tools, machines, type of labour, working conditions, measuring instruments, etc.
3. Quality control can be defined as the entire collection of activities which ensures that the operation will produce the optimum quality products at minimum cost.

4. It can also be defined as the tools, devices or skills through which quality activities are carried out.
5. It is the name of the department which devotes itself full time to quality activities are carried out.
6. The procedure for meeting the quality goals is termed as quality control.
7. It is a system, plan or method of approach to the solution of quality problems.
8. Asper A. Y. Feigenbaum Total Quality Control is:

"An effective system for integrating the quality maintenance and quality improvements efforts of the various groups in an organization, so as to enable production and services at the most economical levels which allow full customer satisfaction".

Steps in Quality Control Programme

1. Formulate quality policy.
2. Work out details of product requirements, set the standards (specifications) on the basis of customer's performance, cost and profit.
3. Select inspection plan and set up procedure for checking.
4. Detect deviations from set standards or specifications.
5. Take corrective action through proper authority and make necessary changes to achieve standards.
6. Decide on salvage method i.e. to decide how the defective parts are disposed of, entire scrap or reworked.
7. Co-ordination of quality problems.
8. Developing quality consciousness in the organization. Quality control is not a function of any single department or a person. It is the primary responsibility of any supervisor to turn out work of acceptable quality.

Aims of Objectives of quality Control

1. To improve the company's income by making the product more acceptable to the customers; by providing long life, greater usefulness (versatility), aesthetic aspects, maintainability, etc.
2. To reduce company's cost through reduction of the losses due to defects. For example, to achieve lower scrap, less rework, less sorting, fewer customer returns etc.
3. To achieve interchangeability of manufacture in large scale production.
4. To produce optimum quality at minimum price.
5. To ensure satisfaction of customer's goodwill, confidence and reputation of manufactures.
6. To make inspection prompt to ensure quality control at proper stages to ensure production of non-defective products.
7. Judging the conformity of the process to the established standards and taking suitable action when there are deviations.
8. To improve quality and productivity by process control, experimentation and customers feedback.
9. Developing procedure for good vendor-vendee relations.
10. Developing quality conscious in the organisation.

Quality Characteristics

A physical or chemical property, a dimension, a temperature, pressure, taste, smell or any other requirements used to define the nature of the product or service (which contributes to fitness for use) is a quality characteristic. Thus, a metal cylinder may be defined by stating the quality characteristic contributes to fitness for use for the product.

Quality characteristic can be classified as:

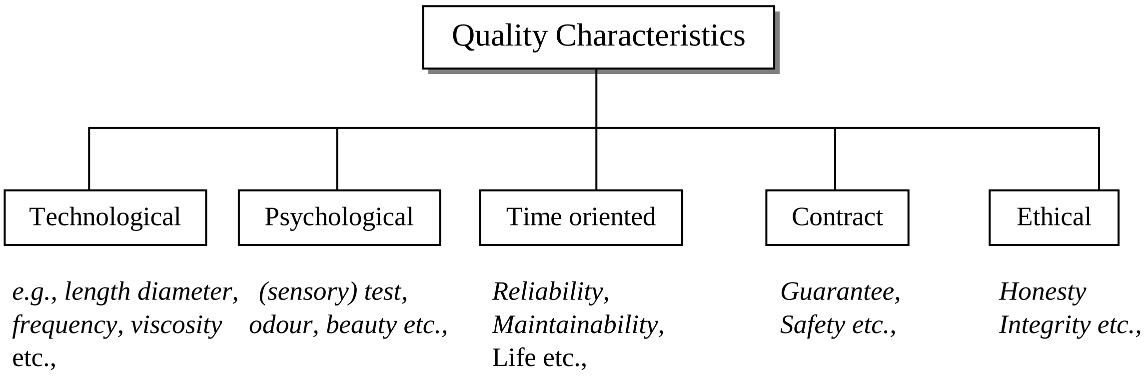

Figure: Quality Characteristics

Quality characteristics may be:

1. Directly measureable example, weight, shear strength, specific gravity, length, diameter etc.
2. Non-measurable example, rejections due to flows, cracks, breakages etc.

For each quality characteristic, there is a sequence of activities as shown in Figure:

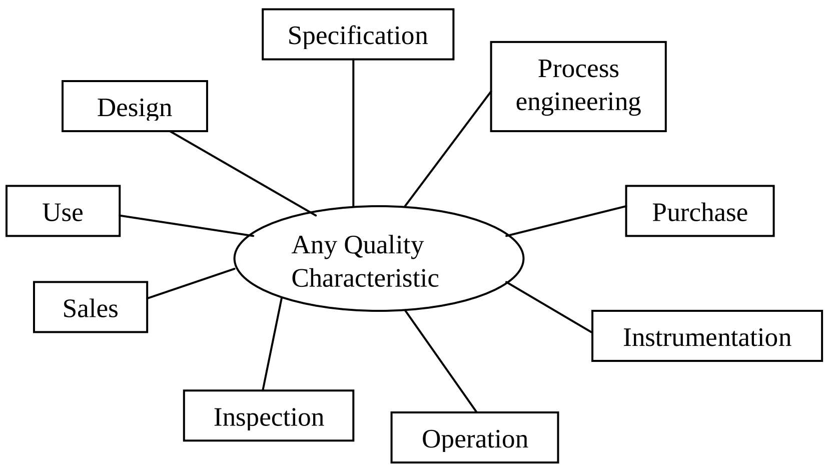

Figure: Sequence of activities for any quality characteristic.

1.2. Basics in Statistical Quality Control (SQC)

A quality control system performs inspection, testing and analysis to ensure that the quality of the products produced is as per the laid down quality standards. It is called “Statistical Quality Control”. The statistical techniques are employed to control, improve and maintain quality or to solve quality problems. Statistics is the collection, organisation, analysis, interpretation and presentation of the data. It is based on law of large numbers and mathematical theory of probability. It is just one of the many tools necessary to solve quality problems it takes into account the existence of variation. Building an information system to satisfy the concept of ‘prevention’ and ‘control’ and improving upon product quality, requires statistical thinking.

SQC is systematic as compared to guess-work of haphazard process inspection and the mathematical statistical approach neutralizes personal bias and uncovers poor judgement. SQC consists of three general activities:

1. Systematic collection and graphic recording of accurate.
2. Analysing the data.
3. Practical engineering or management action, if the information obtained indicates significant deviations from the specified limits.

Modern techniques of SQC and acceptance sampling have an important part to play in the improvement of quality, enhancement of productivity, creation of consumer confidence and development of industrial economy of the country.

Relying itself on probability theory, statistical quality control plays an important role in total quality control. The following statistical tools are generally used for the purpose of exercising control, improvement of quality, enhancement of productivity, creation of consumer confidence and development of the country.

1. Frequency distribution

Frequency distribution is a tabulation or tally of the number of times a given quality characteristic occurs within the samples. Graphic representation of frequency distribution will show:

(a) Average quality
(b) Spread of quality
(c) Comparison with specific requirements
(d) Process capability

2. Control chart

Control chart is a graphical representation of quality characteristics, which indicates whether the process is under control or not.

3. Acceptance sampling

Acceptance sampling is the process of evaluating a portion of the product/material in a lot for the purpose of accepting or rejecting the lot on the basis of conforming or not conforming to a quality specification. It reduces the time and cost of inspection and exerts more effective pressure on quality improvement than it is possibly by 100 percent inspection.

It is used when assurance is desired for the quality of material/products either produced or received.

4. **Analysis of the data**

It includes special methods, which include such techniques as the analysis of tolerance, correlation, analysis of variance, analysis for engineering design, problem solving technique to cause of troubles.

Statistical methods can be used in arriving at proper specification limits of products, in designing the products, in the purchase of raw material, semi-finished and finished products, manufacturing processes, inspection, packaging, sales and also after sales services.

Benefits of Statistical Quality Control

1. **Efficiency:** The use of SQC ensures rapid and efficient inspection at a minimum cost.
2. **Reduction of Scrap:** It uncovers the cause of excessive variability in manufactured produced – forecasting trouble before rejections occur and reducing the amount of spoiled work.
3. Moreover, the use of acceptance sampling in SQC, exerts more effective pressure for quality improvement than is possible by 100% inspection.
4. **Easy detection of faults:** In SQC after plotting the control charts $\overline{X}, R, p, c, u, np$. When the points fall above the upper control limits or below the lower control limit it is an indication of deterioration in quality, necessary corrective action is then taken. On the other hand, with 100% inspection, unwanted variations in quality may be detected at a stage when large amount of defective products have already been produced.
5. **Adherence to specification:** So long as a statistical control continues specifications can be accurately predicted for future, by which it is possible to assess whether the production processes are capable of producing the products with the given set of specifications.
6. Increases, output and reduces wasted machine and man hours.
7. Efficient utilization of personnel, machines and materials resulting in higher productivity.
8. Better customer relations through general improvement in product and higher share of the market.
9. SQC has provided a common language that may be used, by all three groups (designers, production personnel and inspectors) in arriving at a rational solution of mutual problems.
10. Elimination of bottlenecks in the process of manufacturing.
11. Point out when and where 100 percent inspection, sorting or screening is required.
12. Creating quality awareness in employees.

However, it should be emphasized that SQC is not a panacea for assuring product quality. It simply furnished “perspective facts” upon which intelligent management and engineering action can be based. Without such action, the method is ineffective. Even the application of standard procedures without adequate study of the process is extremely dangerous.

Meaning and Scope of Statistical Quality Control

Quality has become one of the most important consumer decision factors in the selection among competing products and services. The traditional definition of quality is based on the viewpoint that products and services must meet the requirements of those who use them, that is customer's risk).

Quality means fitness for use.

Quality is inversely proportional to variability.

There are two general aspects of fitness for use

1. Quality of Design
2. Quality of conformance

All products and services are produced in various in grades or levels of quality are international and consequently, the appropriate technical term in Quality of Design.

For example, all automobiles have as their basis objective providing safe transportation for the consumer. However, automobiles differ with respect to size, appointments, appearance and performance. These differences between the types of automobiles.

The quality of conformance is how well the product conforms to the specifications required by the design. Quality of conformance is influenced by a number of factors, including the choice of manufacturing process, the training and supervision of the workforce, the type of quality assurance system used (process control, tests, inspection activates etc.), the extent to which these quality assurance producers are followed and the modification of the workforce to achieve quality.

Dimensions of Quality

Garvin (1987) provides an eight components or dimensions of quality.

1. Performance
2. Reliability
3. Durability
4. Serviceability
5. Aesthetics
6. Features
7. Perceived Quality
8. Conformance to Standards.

1. **Performance (Will the product do the intended job?)**

Potential customers evaluate a product to determine if it will perform certain specific functions and determine how well it performs then.

2. **Reliability (How often does the product fail?)**

   Complex products, such as many applications, automobiles or airplanes will require some repair over their service life. For industry in which the customer's view of quality is greater impacted by the reliability dimension of quality.

3. **Durability (How long does the product last?)**

   This is the effective service life of the product. Customers want products that perform satisfactory over a long period of time.

4. **Serviceability (How easy is it to repair the products?)**

   There are many industries in which the customer's view of quality is directly influenced by how quickly and economically a repair or routine maintenance activity can be accomplished.

5. **Aesthetics (What does the product look like?)**

   This is the visual appeal of the product, often taking into account factors such as style, colour, shape and other censoring features.

6. **Features (What does the product do?)**

   Usually customers associate high quality with products that have added features.

7. **Perceived Quality (What is the reputation of the company or its product?)**

   Customers ready on the past reputation of the company concerning quality of its products. This reputation is divert influenced by failures of the products. This reputation is visible to the public and by how the customer is treated when a quality related problem with the product is reported.

8. **Conformance to Standards (Is the product made exactly as these designer intended?)**

   A high quality product exactly meets the requirements of customers. Manufactured parts that do not exactly meet the designer's requirements can cause significant quality provides when they are used as the components of a more complex assembly.

Quality Improvement

Quality improvements are the reduction of variability in process and products.

Quality Characteristic

Every product possesses a number of elements that jointly describe what the uses or consumer thinks of as quality. There parameters are often called quality characteristic.

Quality Characteristic may be of several types

1. Physical: length, weight, voltage, viscosity
2. Sensory: taste, appearance, colour
3. Time Orientation: reliability, durability, serviceability

Quality Engineering

Quality engineering is the set of operational, managerial and engineering activities that a company uses to ensure that the quality characteristics of a product are at the nominal of required levels.

A values of a measurements that corresponds to the desired values for that quality characteristic.

Note

Most organisations find it difficult and expressive to provide the customer with products that have quality characteristics that are always identical from unit to unit that match customer’s expectations. A major season for this is variability there is a certain amount of variability in every product consequently. No two products are ever identical. So this variability can be described by some statistical methods. These methods plays a central role inequality improvements efforts.

- In the application of statistical method to quality. Engineering it is fairly typical to classify data on quality characteristic as either attributes or variables.
- Variable data are usually continuous measurements such as length voltage or viscosity.
- Attributes data are usually discrete data often taking the form of counts.

Continuous data involve counts (integer) for example, number of admissions, number of patients waiting, number of defective items.

Upper Specification Limit (USL)

Quality characteristic are often calculate relative to specifications for a manufactured product the specifications are the desired measurements for the quality characteristic on the components in the final product.

The larger allowable value for a quality characteristic is called USL.

Lower Specification Limit (LSL)

The smallest allowable value for a quality characteristic is called LSL.

Non-Conforming Product

A Quality Product is called non-conforming product.

Non-Conformity

A specific type of failure in the product is called a non-conformity.

Defective

A non-conforming product is considered which are non-conforming product is considered defective

Defects

If it has one or more defects, which are non-conformities that are series enough to significantly affect the safe or effective use of the product.

Quality Product

A quality product is defined as a product that meets the needs of the market place.

Quality and Improving Quality

Quality and improving quality has become an important business strategy for many organization, manufactures, distributors, transportation companies, financial services organizations, health care providers and governments agencies.

Non-conforming unit

A non-conforming unit is a unit of product that does not satisfy one or more of the specifications for that product.

Difference between defect and defective

An item is said to be defective if it fails to conform to the specifications in any of the characteristic.

Each characteristic that does not meet the specifications is a defect.

An item is defective if it contains at least one defect. For example, if a casting contains undesirable hard spots, blow holes etc., the casting is defective and the hard spots, blow holes etc. are the defects, which make the casting defective.

The np chart applies to the number of defectives in subgroups of constant size. Whereas c chart applies to the number of defects in a subgroup of constant size.

1.3. Control Chart

A control chart is an important aid or statistical device used for the study and control of the repetitive processes. It was developed by A. Shewhart and it is based upon the fact that variability does exist in all the repetitive process.

A control chart is a graphical representation of the collected information. The information may pertain to measured quality characteristics of samples.

Control Chart patterns

The Control Chart patterns can be classified into two categories

1. Chance pattern of variation (or) Common cause of variation
2. Assignable cause pattern of variation

1. Chance Pattern of Variation

A control chart having a chance pattern of variation will have the following three characteristics:

(i) Most of the points will lie near the central line.
(ii) Very few points will be near the control limits.
(iii) None of the points (except 3 in a thousand) fall outside the control limits.

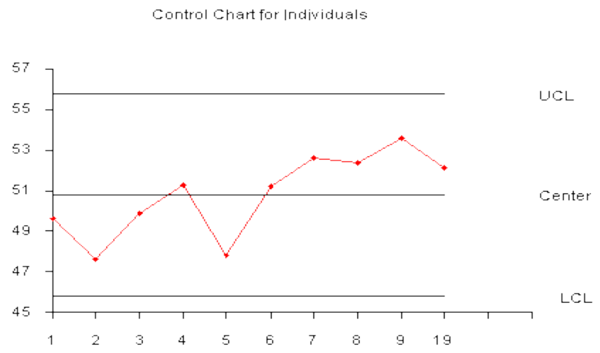

Figure: Control Chart for Chance Pattern of Variation

2. Assignable Cause Pattern of Variation

The most important types of assignable cause patterns of variations are:

(i) Extreme Variation
(ii) Indication of Trend
(iii) Shifts
(iv) Erratic Fluctuations

(i) Extreme Variation

Extreme variation is recognised by the points falling outside the upper and lower control limits. Thus, when the sample points outside these limits on $\overline{X}$ chart, p chart or both it means some assignable causes of error are present and corrective action is necessary to produce the products within the specified limits.

(a) Error in measurement and calculations
(b) Samples chosen at a peak position of temperature, pressure and such other factors.
(c) Wrong setting of machine, tools, etc.
(d) Samples chosen at the commencement or end of an operation.

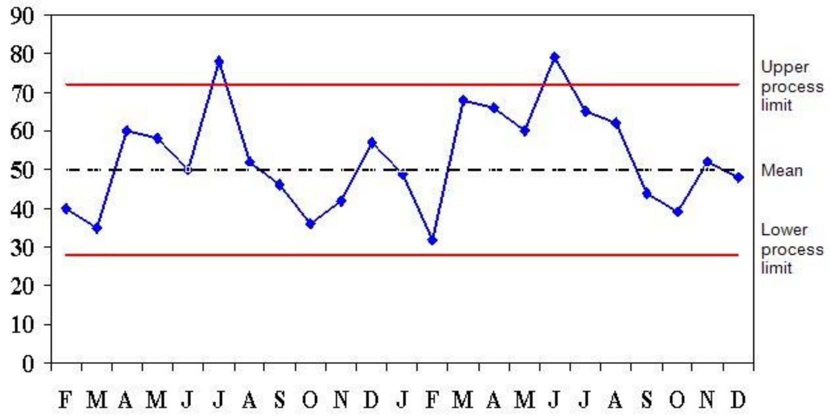

Figure: Control Chart for Extreme Variation

(ii) Indication of Trend

If the consecutive points on $\overline{X}$ or R chart tend to move steadily either towards LCL or UCL, it can be assumed that process is indicating a Trend, that is change is taking place slowly and through all the points are lying with in control limits, after some time it is likely that the process may go out of control if proper case or corrective action is not taken.

Causes of Trend

(i) Tool wear
(ii) Wear of threads on clamping device
(iii) Effects of temperature and humidity
(iv) Accumulation of dirt and clogging of fixtures and holes

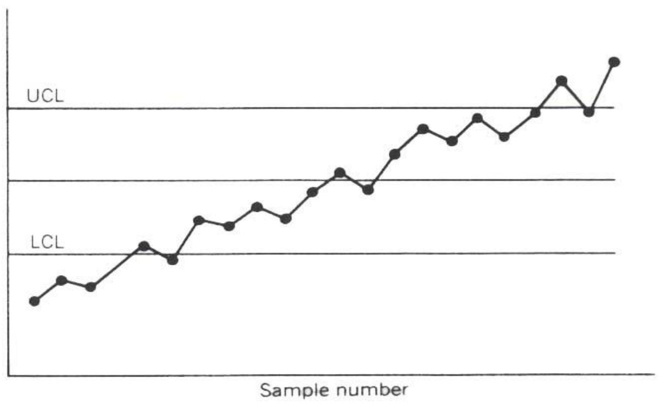

Figure: Control Chart for Indication of Trend

(iii) Shift

When a series of consecutive points fall above or below the central line on either $\overline{X}$ or R chart, it can be assumed that shift in the process has taken place indicating presence of some assignable cause.

It is generally assumed that when seven consecutive points lie above or below the central line, the shift has occurred.

Causes of Shift

- (i) Change in material
- (ii) Change in operator, inspector, inspection equipment
- (iii) Change in machine setting
- (iv) New operator, carelessness of the operator
- (v) Loose fixture etc.

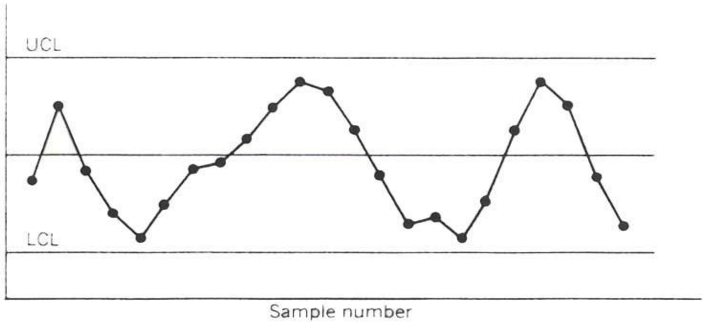

Figure: Control Chart for Shift

(iv) Erratic Fluctuations

Erratic fluctuation is characterised by ups and downs as shown in Figure is given below. This may be due to single causes or a group of causes affecting the process level and spread. The causes of erratic fluctuations are rather difficult to identify. It may be due to different causes acting at different times on the process.

Causes of Fluctuations

- (i) Frequent adjustment of machine
- (ii) Different types of material being processed
- (iii) Change in operator, machine, test equipment etc.

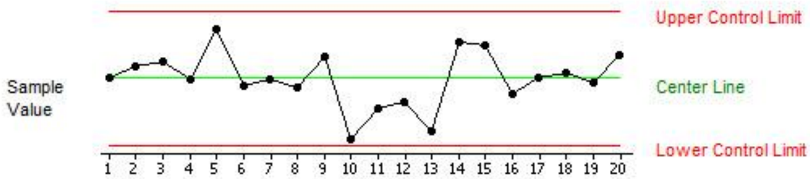

Figure: Control Chart for Erratic Fluctuations

Brief History of Statistical Quality Control

1924: W. A. Shewhart introduces the control chart concept in a Bell Laboratories Technical Memorandum.

1928: Acceptance sampling method is developed and redefined by H. F. Dodge and H. G. Roming at Bell Laboratories.

1944: Industrial quality control (a journal) beings publications.

1946: The American Society for Quality Control (ASQC) is formed as a merger of varies quality societies.

1957: Turan and Gragna: Published first time Quality Control Handbook.

1959: Technometrics (a journal of statistical for the physical, chemical engineering sureness) is established J. Stuart Hunder is the founding editor.

1969: Industrial Quality Control replaced by Quality Progress (Journal of Quality Technology).

1989: The journal Quality Engineering appears.

1989: Motorola's six sigma initiative begins.

1990: ISO 9000 certification activities increase in U. S. Industry and soon.

Basic Principles of Control Chart

A typical control chart is graphical display of a quality characteristic that has been measured or computed from a sample statistic versus the sample number or time. The chart contains center line that represents the average value of the quality characteristic corresponding to the in-control state. Two other horizontal lines called Upper Control Limit (UCL) and Lower Control Limit (LCL). These control limits are chosen so that if the process is in control, nearly all of the sample points will fall between them. As long as the points plot within the control limits, the process is assumed to be in control and no action necessary. If a point that plots outside of the control limits is interpreted as the process is out of control.

Even if all the points plot inside the control limits, if they behave in a systematic or non-random manner, then this could be an indication that the process is out of control. If the process is in control, all the plotted points should have an essentially random pattern. Usually, there is a reason why a particular non-random pattern appears on a control chart and if it can be found and eliminated process performance can be improved.

There is a close connection between control charts and hypothesis testing. If the value of $\overline{X}$ plots between the control limits, we conclude that the process mean is in control that is, it is equal to some value μ_0. On the other hand, if $\overline{X}$ exceeds either control limit, we conclude that the process mean is out of control that is, it is equal to some value $\mu_1 \neq \mu_0$. In a sense, then the control chart is a test of the hypothesis of statistical control and a point plotting outside the control limits is equivalent to rejecting the hypothesis of statistical control. But there are some differences in view point between control charts and hypothesis tests. For example, when testing statistical hypothesis, we usually check the validity of assumptions, whereas control charts are used to detect the shifts.

One place where the hypothesis testing framework is useful is in analysing the performance of control chart. For example, we may think of the probability of type I error of the control chart (concluding the process is out of control when it is really in control) and the probability of type II error of the control chart (concluding the process is in control when it is really in control). It is occasionally helpful to use the operating characteristic curve of a control chart to display its probability of type II error. This would be an indication of the ability of the control chart to detect process shifts of different magnitudes.

In identifying and eliminating assignable causes, it is important to find the underlying root cause of the problem and to attack it.

A very important part of the corrective action process associated with control chart usage is the Out of Control Action Plan (OCAP). The OCAP contains of check points, which are potential assignable causes and terminators, which are actions taken to resolve the out of control condition hopefully by eliminating the assignable cause.

An OCAP is a living document in the sense that it will be modified over time as more knowledge and understanding of the process is gained. Consequently, when a control chart is introduced and initial OCAP should accompany it. Control charts without an OCAP are not likely to be very useful as a process improvement tool.

Control charts may be classified into two general types. Control charts for central tendency and variability are collectively called variable control charts. Many quality characteristics are not measured on a continuous scale or even a quantitative scale. In these cause, we may judge each unit of product as either conforming or non-conforming on the basis count the number of non-conformities. Control charts for such quality characteristic are called attributes control charts.

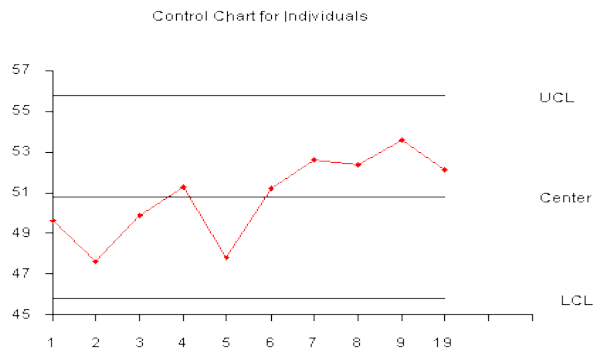

Figure: A typical control chart

Uses of Control Charts

The most important use of a control chart is to improve the process. We have found that, generally,

1. Most processes do not operate in a state of statistical control.

2. Consequently, the routine and attentive use of control charts will identify assignable causes. If these caused can be eliminated from the process, variability will be reduced and the process will be improved.
3. The control chart will only detect assignable causes. Managements, operator and engineering action will usually be necessary to eliminate the assignable causes.

Reasons for the Control Charts are popular in Industries

1. Control charts are a proven technique for improving productivity.
2. Control charts are effective in defect prevention
3. Control charts prevent unnecessary process adjustment
4. Control charts provide diagnostic information
5. Control charts provide information about process capability.

Warning Limits on Control Charts

Some analysts suggest using two sets of limits control charts. The outer limits say, at three sigma are the usual action limits, that is, when a point plots outside of this limit, a search for an assignable cause is made and corrective action is taken if necessary. The inner limits, usually at two sigma are called warning limits.

If one or more points fall between the warning limits and the control limits or very close to the warning limit, we should be suspicious that the process may not be operating properly.

The use of warning limits can increase the sensitivity of the control chart. One of their disadvantage is that they may be confusing to operating personnel.

Sample Size and Sampling Frequency

In designing a control chart, we must specify both sample size to use and the frequency of sampling. In general, larger samples will make it easier to detect small shifts in the process. If the process shift is relatively large, then we use smaller sample sizes.

We must also determine the frequency of sampling. The most desirable situation from the point of view of detecting shifts would be take large samples.

Rational Subgroups

Suppose that we are using a $\overline{X}$ control chart to detect changes in the process mean. Then the rational subgroup concept means that subgroups or samples should be selected so that if assignable causes are present, the chance for differences between subgroups will be maximized, while the chance for differences due to these assignable causes within a subgroup will be minimized.

When control charts are applied to production processes, the time order of production is a logical basis for rational sub grouping. Even though time order is preserved, it is still possible to form subgroups. Time order is frequently a good basis forming subgroups because it allows us to detect assignable causes that occur overtime.

Two general approaches are used to constructing rational subgroups. In the first approach, each sample consists of units that were produced at the same time. Ideally, we would like to take consecutive units of production. This approach is used when the primary purpose of the control chart is to detect process shifts. It minimizes the change of variability due to assignable causes within a sample and it maximizes the change of variability between samples if assignable causes are present. It also provides a better estimate of the standard deviation of the process in the process in the case of variable control charts. This approach to rational sub grouping essentially gives a snapshot of the process at each point in time where a sample is collected.

In the second approach each subgroup is a random sample of all process output over the sampling interval. This method of rational subgrouping is often used when the control chart is employed. In fact, if the process shifts to an out of control state and the back in control again between samples, it is sometimes argued that the first method of rational subgrouping defined above will be ineffective against these types of shifts and so the second method must be used.

When the rational subgroup is a random sample of all units produced over the sampling interval considerable care must be taken in interpreting the control charts. In fact, we can often make any process appear to be in statistical control just by stretching out the interval between observations in the sample.

There are other bases for forming rational subgroups. For example, suppose a process consists of several machines that pool their output into a common stream. If we sample from this common stream of output, it will be very difficult to detect whether or not some of the machines are out of control.

The rational subgroup concept is very important. The proper selection of sample requires careful information of the process, with the objective of obtaining as much useful information as possible from the control chart analysis.

Types of Control Charts

Basically control charts are classified into two types.

1. Variable Control Charts
2. Attribute Control Charts

1.4.Variable Control Chart

Variable control chart mainly consist of three charts namely

1. Mean (Average) control chart ($\bar{X}$)
2. Range control chart (R)
3. Standard deviation control chart (σ)

Attribute Control Chart

Attribute control chart mainly consist of three charts which are

1. Fraction defective chart (p)
2. Chart for defects (c)
3. Chart of number of defectives (np or d)

Assignable Causes of Variation

The assignable causes occurs different situations. They are

1. Points fall outside the control limits
2. A sequence of 7 or more points
3. Points form a trend
4. Points fall very close to central line.

Due to these causes or the presence of assignable causes, the process is considered as out of control.

Constant Cause System or Common Causes

Suppose all the points fall within the control limits but do not form a trend or any sequence of points then it is called constant cause system and the process is in control.

Process Control and Product Control

The main objective in any production process is to control and maintain the quality of the manufactured product. So that it confirms to specified quality standards. This is called process control and it is studied through control charts.

Instead of measuring the products, inspect the product one by one in such a way that, the products will accept or reject as the case may be. This is called product control and it is studied through acceptance sampling plan which has been established by Dodge and Roming.

Specification Limits and Tolerance Limits

Specification Limits

When an article is proposed to be manufactured, the manufactures have to decide upon the maximum and minimum allowable dimensions of some quality characteristics so that the product can be gainfully utilised for which it is intended. If the dimensions are beyond these limits, the product is treated as defective and cannot be used. These maximum and minimum limits of variation of individual items, as mentioned in the product design are known as 'specification limits'.

Tolerance Limits

These are limits of variation of a quality measure of the product between which at least a specified proportion of the product is expected to lie (with a given probability), provided the process is in a state of statistical quality control. For example, we may claim with a probability of 0.99 that at least 90% of the products will have dimensions between some stated limits. These limits are also known as 'statistical tolerance limits'.

The terms specification limits and tolerance limits are often used interchangeably. Indeed, even the American Society for Quality Control's (ASQC) Glossary (1983) defines the two terms with one entry as "the conformance boundaries for an individual unit of a manufacture or service operation". The Glossary does suggest that tolerance limits are generally preferred in evaluating the manufacturing or service environment, whereas specification limits are more appropriate for categorizing materials, products or services in terms of their stated requirements.

For example, a government supply agency provides specifications for mops. One of these specifications concerns the type of wood to be used for the handle. Another specification is the length of the handle as 120±2 cm. The last specification can also be considered a tolerance. Tolerance refer to physical measurements only, whereas specifications refer to characteristics, included in specifications.

Natural process limits are usually determined from population values of from large-sample estimates. Alternatively, a large number of small samples may be collected. In either case, the limits are at $\pm 3\sigma$.

Statistical tolerance limits are not to be confused with tolerance limits. Tolerance limits are set by the designers and appear on engineering drawings. Statistical tolerance limits are based on samples from the production process.

There may be lower tolerance limits (lower specification limits) and upper tolerance limits (upper specification limits). The lower tolerance limit defines the lower conformance boundary for an individual unit of a manufacturing or service operation and the upper tolerance limit applies to the upper conformance boundary.

Finally, since the ASQC Glossary equates tolerance limits to specification limits, lower tolerance limits to lower specification limits, we will also use these terms interchangeably on occasion.

Specifying Tolerances

It is practically impossible to manufacture one article exactly like another or one batch like another. Variability is one of the fundamental concepts of modern quality control. Therefore, the ranges of permissible difference in dimensions have been standardized under the name limits. The limits of size for a dimension or a part are two extreme permissible sizes for that dimension (high limit and low limit).

The difference between the high limit and the low limit which is the margin allowed for variation in workmanship is called tolerance. Tolerance can also be defined as the amount by which the job is allowed to go away from accuracy and perfectness without causing any functional trouble, when assembled with the mating part and put into actual service. Tolerance are set only on dimensions, but also on other quality characteristics as well, such as temperature, pressure and volume.

The selection of tolerance is very important. A common complaint among production personnel is that designers do not understand production problems. Inspection personnel often complain not only about the poor quality of manufactured product but also about the unreasonableness of specified tolerances. Therefore, the designers may specify one tolerance inspection gauges may allow another usually wider tolerance and foreman may be even more liberal.

3σ Limits

A. Shewhart has proposed 3σ limits to construct control charts. The distance between central line and any way one of the control limits is 3σ. Let us take a sample of size n is $(x_1, x_2, \ldots, x_n)$ and its statistic is taken as t. (i.e.) $E(t) = \mu_i$ *and* $V(t) = \sigma_i^2$. Thus,

$$t \sim N(\mu_i, \sigma_i^2)$$

According to normal law,

$$\Rightarrow P[-3\sigma_i \le t - \mu_i \le 3\sigma_i] = 0.9973$$
$$\Rightarrow P[|t - \mu_i| \le 3\sigma_i] = 0.9973$$
or
$$\Rightarrow P[|t - \mu_i| > 3\sigma_i] = 0.0027$$

The above probabilities states that, the probability of a random variable which goes outside 3σ limits $(\mu_i \pm 3\sigma_i)$ is considerably very small. Suppose t follows normal distribution, the observed values lie between $\mu_i + 3\sigma_i$ *and* $\mu_i - 3\sigma_i$ which are called UCL and LCL respectively. If any observed value falls outside the control limits, it is a danger signal for presenting assignable causes.

Construction of Average and Range $(\overline{X}$ *and* $R)$ Charts

The construction of Average and Range charts are based on measurements of produced goods. The measurements may be length, breath, area or volume. The selection of samples or subgroups is very essential. We select N samples in which each sample has n subgroups.

Let X_{ij} be the j^{th} observation of the i^{th} sample (i=1,2,…,N; j=1,2,...n). From the measurable data we have to calculate sample statistics such as mean$(\overline{X}_i)$, Range (R_i) and Standard deviation (S_i) of i^{th} sample,

$$\overline{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{ij} \quad (1)$$

$$R_i = \underset{j}{\text{Max}}\, X_{ij} - \underset{j}{\text{Min}}\, X_{ij} \quad (2)$$

$$s_i = \sqrt{\frac{\sum_{i=1}^{n}(X_{ij} - X_i)^2}{n}} \quad (3)$$

By using above statistics, we have to compute their averages

$$\overline{\overline{X}} = \frac{\sum \overline{X}_i}{N} \quad (4)$$

$$\bar{R} = \frac{1}{N}\sum_{i=1}^{N} R_i \quad (5)$$

$$\bar{s} = \frac{1}{N}\sum_{i=1}^{N} s_i \quad (6)$$

After finding the averages of the statistics, we have to frame the control limits for framing a limits we have to calculate the value of 3σ, where σ is the standard deviations of the universe. The standard error of i^{th} subgroup is defined

$$SE(\bar{X}_i) = \frac{\sigma}{\sqrt{n}}, i = 1,2,\ldots,N \quad (7)$$

From the sampling distribution of range,

$$E(R) = \bar{R} = d_2\sigma \quad (8)$$

$$\Rightarrow \sigma = \frac{\bar{R}}{d_2} \quad (9)$$

where d_2 is a constant relative to the subgroup size.

Control chart for average is constructed when,

(i) μ and σ are unknown
(ii) μ and σ are known
(iii) R is unknown

Case (i): μ and σ are unknown

Suppose the population mean (μ) and population standard deviation (σ) are not given we have to calculate the sample mean and sample standard deviation or sample range.

Let $\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_N$ be averages of subgroups. The average of averages is estimated by using the formula given in equation (4). The required 3σ control limits is defined as

$$E(\bar{X}_i) \pm 3SE(\bar{X}_i) = \bar{\bar{X}} \pm \frac{3\sigma}{\sqrt{n}} \quad \text{(using (7))}$$

$$= \bar{\bar{X}} + \frac{3}{\sqrt{n}}\frac{\bar{R}}{d_2} \quad \text{(using equation (9))}$$

$$= \bar{\bar{X}} + \left(\frac{3}{\sqrt{n}} d_2\right)\bar{R}$$

$$= \bar{\bar{X}} \pm A_2\bar{R} \quad (10)$$

Here A_2 is also a constant which is obtained from a table containing the factors for control charts and it depends on its subgroup size n.

The equation (10) is re-written as

$$UCL = \overline{\overline{X}} + A_2\overline{R} \quad (11)$$

$$LCL = \overline{\overline{X}} - A_2\overline{R} \quad (12)$$

By using equation (4), we draw a horizontal line parallel to x - axis and it represents the central line of the chart. $\therefore CL = \overline{\overline{X}}$. Similarly using equations (11) and (12), we draw dotted horizontal lines and they represent upper control limit and lower control limit respectively.

Using subgroup averages (equation (1)) we plot the points and infer that weather the process is in control or out of control. If the process is out of control do the necessary steps and draw the process is in control.

Case (ii): μ and σ are known

In the case of known population constants we have to calculate sample subgroup means and standard deviation. We have

$$E(\overline{X}_i) \pm 3SE(\overline{X}_i) = \mu \pm 3\frac{\sigma}{\sqrt{n}}$$

$$= \mu \pm \frac{3}{\sqrt{n}}\sigma$$

$$= \mu \pm A\sigma \quad (13)$$

where A is a constant depends on n. The required control limits are,

$$UCL = \mu + A\sigma \quad (14)$$

$$LCL = \mu - A\sigma \quad (15)$$

Case (iii): R is unknown

In the case of unknown range we have to construct $\overline{X}$ chart by using another statistic called standard deviation. The standard deviation of i^{th} subgroup s_i (given in (3)) is calculated and also the average of standard deviation is computed by using

$$\overline{S} = \frac{1}{N}\sum_{i=1}^{N} s_i$$

We know that the relation between average of the sample standard deviations and population standard deviations.

$$\overline{S} = C_2\sigma$$

$$\Rightarrow \sigma = \frac{\overline{S}}{C_2} \quad (16)$$

Control limits are defined as

$$E\left(\bar{X}_i\right) \pm 3SE\left(\bar{X}_i\right)$$

$$= \bar{\bar{X}} \pm 3\frac{\sigma}{\sqrt{n}}$$

$$= \bar{\bar{X}} \pm \frac{3}{\sqrt{n}}\frac{\bar{S}}{C_2}$$

$$= \bar{\bar{X}} \pm \left(\frac{3}{\sqrt{n}C_2}\right)\bar{S}$$

$$= \bar{\bar{X}} \pm A_1\bar{S} \qquad (17)$$

The required control limits are

$$UCL = \bar{\bar{X}} + A_1\bar{s} \qquad (18)$$

$$LCL = \bar{\bar{X}} - A_1\bar{s} \qquad (19)$$

$$CL = \bar{\bar{X}} \qquad (20)$$

here A_1 is also a constant depends on n. By using the value of $\bar{\bar{X}}$, we draw a horizontal line parallel to x-axis and it is named as central line of the chart (20). The upper and lower control limits are drawn are dotted horizontal lines by using the equations (18) and (19) respectively.

The values of subgroup averages are plotted in the chart and verified the process is in control or not.

Control Limits for R Chart

Let X_{ij} be j^{th} observation in i^{th} subgroup (i=1, 2, …, N; j=1, 2, …, n). We have to find range for each subgroup. The range of i^{th} subgroup is,

$$R_i = Max(X_{ij}) - Min(X_{ij}) \quad (i = 1,2,\ldots,N)$$

The average of ranges is

$$\bar{R} = \frac{1}{N}\sum_{i=1}^{N} R_i$$

Control limits for R chart are defined as

$$E(R) \pm 3SE(R)$$

$$= \bar{R} \pm 3C\bar{R}$$

$$= (1 \pm 3C)\bar{R}$$

Here

$$E(R) = \bar{R}$$

$$SE(R) = \sigma R$$

We know that, $\sigma R = C.\ E(R)$

$$= C\overline{R}$$

$$SE(R) = \sigma R = C\overline{R}$$

Hence, the limits are

$$UCL = (1 + 3C)\overline{R} = D_4\overline{R}$$

and

$$LCL = (1 - 3C)\overline{R} = D_3\overline{R}$$

Here D_3 and D_4 are constants taken from the table containing the factors of control charts defending on the subgroup size.

If the subgroup size is less than 7, D_3 becomes zero. In this case we obtain only upper control limit. As in the case of $\overline{X}$ chart, we draw central line by using the value of $\overline{R}$ as a bold horizontal line and the upper control limit is a drawn as dotted horizontal line using in the value of $D_4\overline{R}$. After drawing in central line and control limit we plot the values of ranges. Finally we concluded that the process is in control or not. Suppose a subgroup size $n \geq 7$, lower control limit is also drawn as dotted horizontal line.

Control Chart for Standard Deviation σ Chart

Let X_{ij} be j^{th} observation of the i^{th} sample. Let S and σ be the standard deviations of sample and population. Control chart for standard deviation is constructed under the condition that when σ is unknown and σ is known.

Case (i): σ is unknown

Suppose the population standard deviation is not known, we have to calculate sample standard deviation for constructing standard deviation chart. Let S_i be the standard deviation of i^{th} subgroup

$$S_i = \sqrt{\frac{\sum (X_{ij} - \overline{X}_i)^2}{n}}$$

The average of the standard deviation is obtained as

$$\overline{S} = \frac{1}{N}\sum_{i=1}^{N} S_i$$

We know that, the relation between population standard deviation and average of the sample standard deviation is

$$\overline{S} = C_2\sigma$$

The control limits of standard deviation chart are

$$= E(S) \pm 3SE(S)$$

$$= \bar{S} \pm 3(C_3\sigma)$$

$$= \bar{S} \pm 3C_3\left(\frac{\bar{S}}{C_2}\right)$$

$$= \bar{S} \pm \left(3\frac{C_3}{C_2}\right)\bar{S}$$

Hence,

$$UCL = \left(1 + 3\frac{C_3}{C_2}\right)\bar{S} = B_4\bar{S}$$

$$LCL = \left(1 - 3\frac{C_3}{C_2}\right)\bar{S} = B_3\bar{S}$$

After drawing central line and control limits, plot the points by using subgroup standard deviation and draw suitable conclusion. If the value of lower control limit is negative, we take the value as zero. Here the values of D_3 and D_4 are taken from the pre-assigned table depending on subgroup size n.

Case (ii): σ is known

Suppose the population standard deviation is known, there is no need to compute sample statistic, now we define the following:

$E(S) = C_2\sigma$ and

$SE(S) = C_3\sigma$.

The control limits of standard deviation chart are

$$= E(S) \pm 3SE(S)$$

$$= C_2\sigma \pm 3C_3\sigma$$

$$= (C_2 \pm 3C_3)\sigma$$

Hence,

$$UCL = (C_2 + 3C_3)\sigma = B_2\sigma$$

$$LCL = (C_2 - 3C_3)\sigma = B_1\sigma$$

$$Central\ Line = C_2\sigma.$$

Problem 1:

Control charts for $\overline{X}$ and R are maintained on certain dimensions of a manufactured part, measured in mm. The subgroup size is 4. The values of $\overline{X}$ and R are computed for each subgroup. After 20 subgroups $\sum \overline{X} = 412.83 \, and \sum R = 3.39$. Compute the values 3 sigma limits for the $\overline{X}$ and R charts and estimate the value of σ' on the assumption that the process is in statistical control.

Solution:

$$\overline{\overline{X}} = \frac{\sum \overline{X}}{N}$$

Where N=number of subgroups

Therefore,

$$\overline{\overline{X}} = \frac{412.83}{20} = 20.6415$$

$$\overline{R} = \frac{\sum R}{N}$$

$$\overline{R} = \frac{3.39}{20} = 0.169$$

i.e. σ' =population standard deviation

$$\sigma' = \frac{\overline{R}}{d_2} = \frac{0.169}{2.059} = 0.082 \quad \text{[for subgroup of factor } d_2=2.059\text{]}$$

$$3\sigma_{\overline{X}} = \frac{3\sigma'}{\sqrt{n}} = \frac{3 \times 0.082}{\sqrt{4}} = 0.123$$

For $\overline{\overline{X}}$ chart:

$$UCL_{\overline{X}} = \overline{\overline{X}} + 3\sigma_{\overline{X}}$$

= 20.6415+0.123

= 20.7645

$$LCL_{\overline{X}} = \overline{\overline{X}} - 3\sigma_{\overline{X}}$$

= 206415-0.123

= 20.5185

For R chart:

$$UCL_R = D_4 \overline{R}$$

= 2.28*0.169 [for subgroup of 4 factor D_4=2.28 from table]

= 0.3853

$$LCL_R = D_3\overline{R}$$

= 0*0.169 [for subgroup of 4, D_3=0]

= 0.

Problem 2:

In a capability study of a lathe used in turning a shaft to a diameter of 23.75±0.1 mm a sample of 6 consecutive pieces was taken each day for 8 days. The diameters of these shafts are as given below:

1st Day	2nd Day	3rd Day	4th Day	5th Day	6th Day	7th Day	8th Day
23.77	23.80	23.77	23.79	23.75	23.78	23.76	23.76
23.80	23.78	23.78	23.76	23.78	23.76	23.78	23.79
23.78	23.76	23.77	23.79	23.78	23.73	23.75	23.77
23.73	23.70	23.77	23.74	23.77	23.76	23.76	23.72
23.76	23.81	23.80	23.82	23.76	23.74	23.81	23.78
23.75	23.77	23.74	23.76	23.79	23.78	23.80	23.78

Construct the $\overline{X}$ and R chart and find out the process capability for the machine.

Solution:

Average diameter for the first day

$$\overline{X}_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5 + X_6}{6}$$

$$= \frac{23.77 + 23.80 + 23.78 + 23.73 + 23.76 + 23.75}{6} = 23.765$$

Similarly, the average for each day are calculated and the results are tabulated as below:

$\overline{X}_1$	$\overline{X}_2$	$\overline{X}_3$	$\overline{X}_4$	$\overline{X}_5$	$\overline{X}_6$	$\overline{X}_7$	$\overline{X}_8$
23.765	23.77	23.7716	23.7767	23.7717	23.7583	23.7767	23.7667

Now,

$$\overline{\overline{X}} = \frac{\sum \overline{X}}{N}$$

$$= \frac{190.1567}{8} = 23.7696.$$

Ranges:

R_1	**R_2**	**R_3**	**R_4**	**R_5**	**R_6**	**R_7**	**R_8**
0.07	0.11	0.06	0.08	0.04	0.05	0.06	0.07

$$\overline{R} = \frac{\sum R}{N} = 0.0675$$

For $\overline{X}$ chart:

$$UCL_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R}$$

= 23.7696+0.48*0.0675 [A_2 = 0.48 for subgroup of from table appendix]

= 23.802

$$LCL_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R}$$

= 23.7696-0.0324

= 23.7322

For R chart:

$$UCL_R = D_4\overline{R}$$

= 2*0.0675

= 0.1350

$LCL_R = D_3\overline{R} = 0$ [D_3=0 for subgroup of 6 or less]

Process capability:

$$6\sigma' = 6 \times \frac{\overline{R}}{d_2} = \frac{6 \times 0.0675}{2.534} = 0.15982 \quad [\textit{for subgroup of } 6, d_2 = 2.534 \textit{ from table appendix}]$$

$X_{\max} - X_{\min} = 0.2\,mm$ from data.

Therefore, $(X_{\max} - X_{\min}) > 6\sigma'$.

So, we conclude that all manufactured products will meet specifications as long as the process stays in control.

Problem 3:

The following table shows the average and ranges of the spindle diameters in millimetres for 30 subgroups of 5 items each.

$\overline{X}$	R	$\overline{X}$	R	$\overline{X}$	R
45.020	0.375	45.600	0.275	45.26	0.150
44.950	0.450	45.020	0.175	45.650	0.200
45.480	0.450	45.320	0.200	45.620	0.400
45.320	0.150	45.560	0.425	45.480	0.225
45.280	0.200	45.140	0.250	45.380	0.125
45.820	0.250	45.620	0.375	45.660	0.350
45.580	0.275	45.800	0.475	45.460	0.225
45.400	0.475	45.500	0.200	45.640	0.375
45.660	0.475	45.780	0.275	45.390	0.650
45.680	0.275	45.640	0.225	45.290	0.350

For the first 20 samples set up on $\overline{X}$ chart and R an R chart. Plot the next 10 samples on these charts to see if the process continues "under control" both as to average and range. Also find the process capability.

Solution:

$$\overline{\overline{X}} = \frac{\sum \overline{X}}{N} = \frac{909.170}{20} = 45.4585$$

$$\overline{R} = \frac{\sum R}{N} = \frac{6.250}{20} = 0.3125$$

$$\sigma' = \frac{\overline{R}}{d_2} = \frac{0.3125}{2.326} = 0.13435$$

$$\sigma_{\overline{X}} = \frac{\sigma'}{\sqrt{n}} = \frac{0.13435}{\sqrt{5}} = 0.06009$$

$$UCL_{\overline{X}} = \overline{\overline{X}} + 3\sigma_{\overline{X}}$$

= 45.4585+3*0.06009

= 45.4585+0.1803

= 45.6388

$$LCL_{\overline{X}} = \overline{\overline{X}} - 3\sigma_{\overline{X}}$$

= 45.4585-0.1803

= 45.2782

$$UCL_R = D_3\overline{R}$$

= 2.11*0.3125

= 0.6594

$$LCL_R = D_3\overline{R} = 0$$

Process Capability:

$$6\sigma' = 6 \times 0.13425 = 0.80550$$

Exercises

1. Define statistical quality control.
2. What are the causes of quality variation?
3. What are the applications of control chart?
4. A sub-group of 5 items each are taken from a manufacturing process at a regular interval. A certain quality characteristic is measured and $\overline{X}$ and R values computed. After 25 subgroups it is found that $\sum \overline{X} = 357.50 \, and \sum R = 8.80$. If the specification limits are 14.40±0.40 and if the process is in statistical control, what conclusions can you draw about the ability of the process to produce items within specifications? (For subgroup of 5 items, $d_2 = 2.326$).
5. Explain the meaning and scope of Statistical Quality Control.
6. Explain (i) 3σ limits (ii) control limits.
7. Explain the construction and operations of $\overline{X}$ and R chart.
8. Explain the concept of rational of rational subgroups, specification, tolerance and warning limits.
9. Determine the control limits for $\overline{X}$ and R charts if $\sum \overline{X} = 357.50 \, and \sum R = 9.90$, Number of subgroups = 20. It is given that $A_2 = 0.18, D_3 = 0.41$, $D_4 = 1.59$ and $d_2 = 3.735$. Also find the process capability.
10. (a) Differentiate between the Chance causes and Assignable causes of variation giving suitable examples.

    (b) What is meant by natural tolerance of process?
11. Explain the factors to be considered in determining

    (a) Sample size.

    (b) Frequency of subgrouping.

    (c) Basis of subgrouping.

Unit II

Control Charts for Attributes

2.1. Introduction

2.2. Control chart for fraction defectives (p – chart)

2.3. Control chart for number of defectives (np or d – chart)

2.4. Control chart for number of defects per unit (c – chart)

2.5. Control chart for number of defects in variable sample size (u – chart)

2.1. Introduction

Average and Range charts are very powerful statistical techniques to point out the troubles in the production process. Variable charts are drawing based on measurable units. They do not help to study the quality characteristics of the products. For analysing the quality characteristics of the products, Shewhart has established another set of control charts which are called attribute control charts and they are given below:

1. p – chart: control chart for fraction defective
2. np – chart: control chart for number of defectives
3. c – chart: control chart for number of defects
4. u – chart: control chart for number of defects in variable sample size.

2.2. Control chart for fraction defectives (p – chart)

In production process, inspection is carried out for identified conformity and non-conformity units. Let d be the number of non-conformity is a sample of size n. Let p be the sample proportional defective and it is defined as the ratio of the number of non-conformity units to the total number of units inspected. That is,

$$P = \frac{d}{n}$$

The corresponding population proportion is taken as P according to Binomial law the number of non-conformities,

$$d \sim B(n, P)$$
$$\Rightarrow E(d) = nP$$

and v(d) = nPQ, where Q = 1-P for constructing control limits,

$$\mathrm{v(p)} = \frac{1}{\mathrm{n}^2}\mathrm{v(d)}$$

$$= \frac{1}{n^2} nPQ$$

$$v(p) = \frac{PQ}{n}$$

Control limits are defined as

$$E(p) = E\left(\frac{d}{n}\right)$$

$$= \frac{1}{n} E(d)$$

$$= \frac{1}{n} np$$

$$E(p) = P$$

$$E(p) \pm 3SE(p)$$

$$= P \pm 3\sqrt{\frac{PQ}{n}}$$

$$= P \pm A\sqrt{PQ}, \quad \text{where} \quad A = \frac{3}{\sqrt{n}}$$

Case (i): Standards are known

Let P' be the value of P then the control limits are,

$$P' \pm A\sqrt{P'(1-P')} \ \ and \ \ CL = P'$$

Case (ii): Standards are unknown

(a) Sample size varies:

Consider k samples of different sizes $n_1, n_2, \ldots, n_k$ the sample units are inspected and the number of defective units are obtained as $d_1, d_2, \ldots, d_k$ respectively. The fraction defective is defined as,

$$P_i = \frac{d_i}{n_i}, \quad i = 1, 2, \ldots, k$$

The average with the fraction defectives in computed as

$$\overline{P} = \frac{\sum p_i}{k}$$

In other words, $\overline{p}$ is computed as,

$$\bar{p} = \frac{\sum d_i}{\sum n_i} = \frac{\sum n_i p_i}{\sum n_i}$$

$\bar{p}$ indicates the central line. The control limits are

$$\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n_i}$$

Plot the value of p_i in the chart and in found that the process is in control or not. If any point falls outside the control limits, remove that point and construct revised control chart for the remaining observations.

(b) Sample size is fixed

Consider k samples of equal size n inspect each and every sample and the defective units are denoted as $d_1, d_2, \ldots, d_k$ the fraction defective of ith sample defined as the ratio of the number of defective units in the ith sample (d_i) to the sample size.

$$p_i = \frac{d_i}{n}, \; i = 1, 2, \ldots, k$$

The average of the fraction defectives is computed by using the relation

$$\bar{p} = \frac{\sum d_i}{k(n)}$$

$\bar{p}$ indicates the central line. Then the control limits are,

$$\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n}$$

After drawing the control limits and central line, we have to plot the values of p_i in the chart and draw the conclusion whether the process is in control or not. Suppose any point falls outside the control limits, we remove that points and construct the revised control chart.

2.3. Control chart for number of defectives (np or d – chart)

In a production process, the number of non-conformities are collected after expecting the sampling units. Let d be the number of defective units. It is noted that the proportion of defectives is given by the relation p = d/n, this implies that,

$$E(p) = P$$

The mean and variance of non-conformities units for obtained in terms of population proportion

$$\text{Mean} = E(d) = nP$$

$$\text{Variance} = v(d) = nP(1-P)$$

$$\Rightarrow SE(d) = \sqrt{nP(1-P)}$$

Hence, central line = nP.

Control limits are

$$E(d) \pm 3SE(d)$$

$$= nP \pm 3\sqrt{nP(1-P)}$$

$$\Rightarrow UCL = nP + 3\sqrt{nP(1-P)}$$

$$LCL = nP - 3\sqrt{nP(1-P)}$$

After drawing central line and control limits, we have to plot the values of number of defectives (d_i). Suppose any point falls outside the limits, we have to find out the reasons and rectified the causes.

2.4. Control chart for number of defects per unit (c – chart)

In any production process inspections is carried out for supporting standard items and bad items (defective items) by considering defectives, p chart and np chart are appropriate charts.

Suppose one may try to study about the defects per unit, another control chart, namely c chart is used. According to probability law, number of defects per unit follows Poisson distribution. Generally the population average for number of defects is denoted as C.

$$X \sim P(C)$$

$$P(x) = e^{-C} \frac{C^x}{x!}$$

Here, we known that X is the number of non-conformities and C is the parameter.

Case (i): Standards known

Let C be the average of number of defects in the population. C is a Poisson parameter and the control limits are defined as,

$$C \pm 3\sqrt{C}$$

C indicates the central line.

Case (ii): Standards unknown

From industrial products, collect defective pieces. Inspect each and every defective pieces and noted the number of defects per product. Let $c_1, c_2, \ldots, c_n$ be the number of defects and its average is

$$\overline{C} = \frac{\sum_{i=1}^{n} C_i}{n}$$

This $\overline{C}$ is also follows Poisson distribution and it represents the central line of the chart. We required control limits are

$$\overline{C} \pm 3\sqrt{\overline{C}}$$

After drawing central line and control limits, plot the values of $c_1, c_2, \ldots, c_n$ and conclude that the process is in control or not.

2.5. Control chart for number of defects in variable sample size (u – chart)

This control chart is different from c chart. In c - chart, each and every unit is taken as a sample. Suppose we have many number of defective units. There is no possibility to construct the c-chart, we apply u chart, which is otherwise called c chart for variable size.

Let $n_1, n_2, \ldots, n_k$ be sizes of different samples (or) the sampling units are considered as defective units. Observe the number of defective in each unit and count the total defects in each sample. Let $c_1, c_2, \ldots, c_k$ be the number of defects of the above samples $n_1, n_2, \ldots, n_k$ respectively. The ratio of the number of defects (C_i) to the number of defective units (n_i) is taken as,

$$u_i = \frac{c_i}{n_i}, \quad i = 1, 2, \ldots, k$$

Now compute the average of $u_i{}' s$

$$\bar{u} = \frac{\sum_{i=1}^{k} u_i}{k}$$

It is noted that $\bar{u}$ follows Poisson distribution. According to a standard error of sample mean, we have

$$SE(\bar{u}) = \sqrt{\frac{\bar{u}}{n_i}}$$

The control limits are defined as

$$E(u_i) \pm 3SE(u_i)$$

$$\bar{u} \pm 3\sqrt{\frac{\bar{u}}{n_i}}$$

Here, $\bar{u}$ indicates the central line. After drawing control limits, plot the values of $u_i^{\prime}s$ corresponding to sample numbers.

Comparison of $\overline{X}$ and R chart with p chart

1. p chart is attribute control chart, i.e. for quality characteristic that can be classified as either conforming or nonconforming to the specifications. For example, dimensions checked by Go-No-Go gauges. Whereas, $\overline{X}$ and R chart is used for quality characteristic that can be measured and expressed in numbers.
2. The cost of collecting the data for p chart is less than the cost of collecting the data for $\overline{X}$ and R chart. For example, 10 shafts might be inspected with "go-no-go" gauge in the time required to measure a single shaft diameter with a micrometer. Secondly, p chart uses data already collected for other purpose.
3. The cost of computing and changing may also be less since p chart can be applied to any number of quality characteristics observed on one article. But separate $\overline{X}$ and R chart is required for each measured quality characteristic, which may be impracticable and uneconomical.
4. p chart is best suited in cases where inspection is carried out with a view to classifying an article as accepted or rejected. $\overline{X}$ and R charts are best suited for critical dimensions.
5. p chart though discloses the presence of assignable causes of variations, it is not as sensitive as $\overline{X}$ and R chart. For actual diagnosis of causes of troubles, $\overline{X}$ and R charts are best, still p chart can be used effectively in the improvement of quality.
6. The sample size is generally larger for p chart than for $\overline{X}$ and R chart. The variations in the sample size influence the control limits much more in $\overline{X}$ and R charts than in p chart.
7. The control chart for fractions defective provides management with a useful record of quality history.

Purpose of the p- chart

Because of the lower inspection and maintenance costs of p charts, they usually have a greater area of economical applications than do the control charts for variables. A control chart for fraction defective may have any one or all of the following purposes:

1. To discover the average proportion of defective articles submitted for inspection, over a period of time.
2. To bring to the attention of the management, any changes in average quality level.
3. To discover, identify and correct causes of bad quality.
4. To discover, identify and correct the erratic causes of quality improvement.
5. To suggest where it is necessary to use $\overline{X}$ and R charts to diagnose quality problems.
6. In a sampling inspection of large lots of purchased articles.

Basis of control limits on c- chart

Control limits on c chart are based on Poisson distribution. Therefore, two conditions must be satisfied.

- The first condition specifies that the area of opportunity for occurrence of defects should be fairly constant from period. The expression may be in terms of defects per unit being employed. For example, while inspecting the imperfections of a cloth it is necessary to take some units area say 100 square meters and count the number of imperfections per unit (i.e. per 100% square meters). Another example, may be number of point's imperfections per square area of painted surface. However, c chart need not be restricted to a single type of defect but may be applicable for the total of many different kinds of defects observed on any unit.
- Second condition specifies that opportunities for defects are large, while the changes of a defect occurring in anyone spot are small. For example, consider a case in which the product is large unit, say a ratio, which can have defects at number of points although any one point has only few defects.

Comparison between attribute charts and variable charts

Choosing a particular type of chart is a question of balancing the cost of collecting and analysing the type of data required to plot the chart against usefulness of the conclusions that can be drawn from the chart.

	Variable Charts	Attribute Charts
1.	**Example** $\overline{X}$, R, σ charts.	p, np, c, u charts.
2.	**Type of Data Required** Variables data (Measured values of characteristics).	Attribute data (using Go-No-Go gauges).
3.	**Filed of Application** Control of individual characteristics.	Control of proportion of defectives or number of defects or number of defects per unit.
4.	**Advantages** ➢ Provides maximum utilisation of information available from data. ➢ Provides detailed information on process average and variation for control of individual dimensions.	➢ Data required are often already available from inspection records. ➢ Easily understood by all persons. Since, it is more simple as compared to $\overline{X}$ and R chart. ➢ It provides overall picture of quality history.
5.	**Disadvantages** ➢ They are not easily understood unless training is provided. ➢ Can be confusion between control limits and specification limits.	➢ They do not provide detailed information for control of individual characteristic. ➢ They do not recognise different degree of defectiveness.

	➢ Cannot be used with go-no-go gauge inspection.	(Weightage of defects).

Generally compromise is made, it is usual to start with p chart and only for those cases shown out of control on p chart, $\overline{X}$ and R chart are plotted for detailed analysis.

Problem 1:

Following are the inspection results of magnets for nineteen observations

Week Number	Number of magnets inspection	Number of defective magnets	Fraction defective
1	724	48	0.066
2	763	83	0.109
3	748	70	0.094
4	748	85	0.114
5	724	45	0.062
6	727	56	0.077
7	726	48	0.066
8	719	67	0.093
9	759	37	0.049
10	745	52	0.070
11	736	47	0.064
12	739	50	0.068
13	723	47	0.065
14	748	57	0.076
15	770	51	0.066
16	756	71	0.094
17	719	53	0.074
18	757	34	0.045
19	760	29	0.038
Total	14091	1030	

Calculate the average fraction defective and 3 sigma control limits, construct the control chart and state whether the process is in statistical control.

Solution:

The average sample size

$$= \frac{14091}{19} = 741.63 \approx 742$$

The average fraction defectives

$$\overline{p} = \frac{\text{Total defectives in all samples}}{\text{Total inspected in all samples}} = \frac{1030}{14091} = 0.0731$$

$$UCL_p = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n}$$

$$= 0.0731 + 3\sqrt{0.0731(1-0.00731)/742}$$

$$= 0.1018$$

$$LCL_p = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n}$$

$$= 0.0731\text{-}0.0287$$

$$= 0.0444$$

We concluded that from the resulting control chart, sample numbers 2nd and 4th are going above the upper control limits and the sample number 19th goes below the lower control limit. Therefore the process does not exhibit statistical control.

Problem 2:

A certain product is given 100% inspection as it is manufactured and the resultant data are summarized by the hour. In the following table, 156 hours of data are recorded. Calculate the control limits using 3 sigma control limits and indicate the values that are out of control.

Hour	Number of units inspected	Number of defective units	Fraction defective
1	48	5	0.104
2	36	5	0.139
3	50	0	0.000
4	47	5	0.106
5	48	0	0.000
6	54	3	0.0555
7	50	0	0.000
8	42	1	0.0239
9	32	5	0.156
10	40	2	0.050
11	47	2	0.0425
12	47	4	0.085
13	46	1	0.0217
14	46	0	0.000
15	48	3	0.00625
16	39	0	0.000
Total	720	36	

Solution:

The average sample size

$$= \frac{720}{16} = 45$$

The average fraction defectives

$$\bar{\text{p}} = \frac{\text{Total defectives in all samples}}{\text{Total inspected in all samples}} = \frac{36}{720} = 0.05$$

$$UCL_p = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n}$$

$$= 0.05 + 3\sqrt{\frac{0.05(1-0.05)}{45}}$$

$$= 0.14747$$

$$LCL_p = 0.05 - 0.09747$$

$$= -0.04747 = 0$$

Reading number 9 goes out of control. Therefore, the process does not exhibit statistical control.

Problem 3:

A manufacturer purchases small bolts in cartons that usually contain several thousand bolts. Each shipment consists of a number of cartons. As a part of the acceptance procedure for these bolts, 400 bolts are selected at random from each carton and are subjected to visual inspection for certain defects. In a shipment of 10 cartons the respective percentage of defectives in the samples from each carton are 0, 0, 0.5, 0.75, 0, 2.0, 0.25, 0, 0.25 and 1.25. Does this shipment of bolts appear to exhibit statistical control with respect to the quality characteristics examined in the inspection?

Solution:

Average fraction defective

$$\bar{p} = \frac{\text{Total number of defectives}}{\text{Total number inspected}}$$

Therefore $\bar{p}$

$$= \frac{(0+0+0.5+0.75+0+2.0+0.25+0+0.25+1.25) \times \frac{400}{100}}{400 \times 10} = 0.005$$

$$UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.005 + 3\sqrt{\frac{0.005 \times 0.995}{400}}$$

$$= 0.015580$$

$$LCL_p = 0.005 - 3\sqrt{\frac{0.005 \times 0.995}{400}}$$

$$= -0.00558 = 0$$

Since it is never possible to obtain a negative population of defectives, the lower control limit is taken as zero.

After comparing the reading with UCL_p *and* LCL_p it is found that reading number $6 = 2 \times \frac{1}{100} = 0.02$ falls outside the upper control limit. Hence the shipment does not exhibit statistical control.

Exercises

1. In a manufacturing process, the number of defectives found in the inspection of 15 lots of 400 items each are given below:

Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of defectives	2	5	0	14	3	0	1	0	18	8	6	0	3	0	6

   (a) Determine the trial control limits for np chart and state whether the process is in control.
   (b) What will be new value of mean fraction defective if some obvious points outside control limit are eliminated? What will be the corresponding upper and lower control limits? Examine whether the process is still in control or not.

2. The following table gives the number of defectives for lot number ten goes out of control.

Air plane Number	**Number of missing rivets**	**Air plane Number**	**Number of missing rivets**	**Air plane Number**	**Number of missing rivets**
1	8	10	12	19	11
2	16	11	23	20	9
3	14	12	16	21	10
4	19	13	9	22	22
5	11	14	25	23	7
6	15	15	15	24	28
7	8	16	9	25	9
8	11	17	9		
9	21	18	14		

Find $\bar{c}$ compute trail control limits and plot control chart for c. What values of c would you suggest for the subsequent period?

3. What is difference between a defect and defective?
4. Outline the theory underlying control chart for defects.

5. How will you classified defects? Explain.
6. How do you compare p chart with $\overline{X}$ and R chart?
7. Write short notes on:
   (i). Purpose of p chart
   (ii). Basis for control limits on c chart
   (iii). Classification of defects
   (iv). Comparison between attribute control charts and variable control charts.
8. Explain the construction and operations of np chart and p chart.
9. Explain the concept of c chart and u chart.

Operating Characteristic curve for Control Charts (OC Curve)

Control charts are used to detect the shifts in advance. OC curves for control charts will help to detect the shifts.

Consider sample averages $\bar{x}_1, \bar{x}_2, ..\bar{x}_n$ and their common average is taken as $\mu_0 = \bar{x}$. We know that, standard error of $\bar{x} = \frac{\sigma}{\sqrt{n}}$. It is also noted that the distance between central line at $\mu_0 = \bar{x}$ and any control limit is given by $L\frac{\sigma}{\sqrt{n}}$

That is,

$$UCL = \mu_0 + L\frac{\sigma}{\sqrt{n}}$$

$$LCL = \mu_0 - L\frac{\sigma}{\sqrt{n}}$$

$$CL = L\frac{\sigma}{\sqrt{n}}.$$

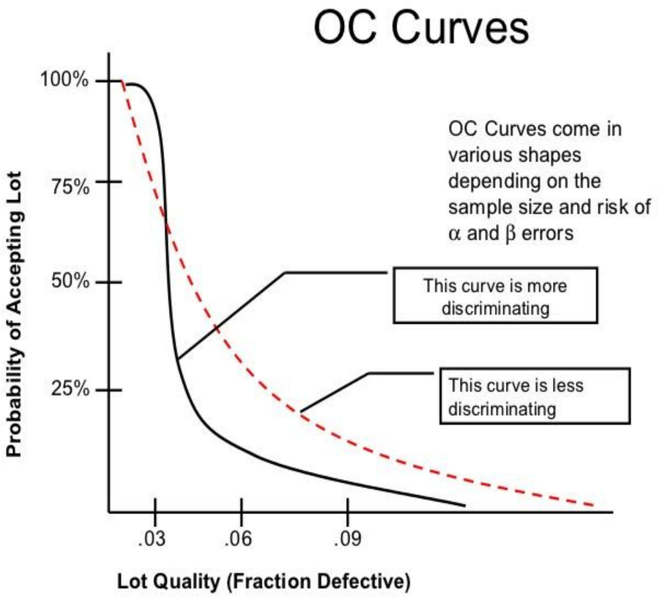

Figure: OC Curve for Control Chart

Let μ_1 be a point which falls outside the control limit. This point is to be detected by using OC function. Now we define $\mu_1 = \mu_0 + k\sigma$.

Let β be the probability of not detecting the shifts.

That is, $$\beta = P\left[LCL \le \bar{x} \le UCL\right]$$

$$= P\left[\frac{LCL - \mu_1}{\sigma/\sqrt{n}} \le \bar{x} \le \frac{UCL - \mu_1}{\sigma/\sqrt{n}}\right]$$

$$\beta = F\left(L - k\sqrt{n}\right) - F\left(-L - k\sqrt{n}\right)$$

Here 1 - β is the probability of detective the shifts and its reciprocal is called average run length

$$ARL = \frac{1}{1-\beta}.$$

Modified Control Limits (Rejection Control Limits)

If any control chart 3σ limit plays a vital role. If μ and σ are process mean and process standard deviation respectively, then the limits $\mu \pm 3\sigma$ are called natural tolerance limits. The overall width 6σ is named as natural tolerance which means that if any value falls outside 6σ level, assignable causes are present then the process may be rejected or revised. Consider the maximum and minimum values from the given observations and X_{max} and X_{min} denote USL and LSL respectively for some quality characteristic. The relationship between natural tolerance limits and specification limits are given below.

i) USL – LSL > 6σ

ii) USL – LSL ≈ 6σ

iii) USL – LSL < 6σ

iv) Among these three relations, if the first relation USL – LSL > 6σ holds good, then modified control limits exhibit the relation between specification limits and the values of $\bar{x}$ in average chart.

If the universe is at the highest accepting position, the process average (central line) will be at a distance 3σ below USL and when the universe is at its lowest accepting position, the process average is at a distance 3σ above LSL.

Similarly the distance between central line and upper control limit (or) lower control limit is $\frac{3\sigma}{\sqrt{n}}$.

For detailed study we have to form a central band instead of fixing central line at $\bar{\bar{x}}$. The upper and lower edges of the central band are defined as

Upper edge, UE = USL – 3σ

Lower edge, LE = LSL + 3σ

The below chart reduced that the highest and lowest value of upper control limit and lower control limit are identical with upper rejection limit and lower rejection limit respectively. On the basic of the diagram, rejection limits are defined as

$$URL = UE + \frac{3\sigma}{\sqrt{n}}$$

$$LRL = LE - \frac{3\sigma}{\sqrt{n}}$$

These equations are rewritten by using the expressions for UE and LE are follows

$$URL = UCL - 3\sigma + \frac{3\sigma}{\sqrt{n}}$$

$$LRL = LCL + 3\sigma - \frac{3\sigma}{\sqrt{n}}$$

These rejection limits are called modified control limits.

Applications of c-chart

The universal nature of Poisson distribution as the law of small numbers makes the c-chart technique quite useful. In spite of the limited field of application of c-chart (as compared to $\overline{X}$, R and p charts), there do exist situation in industry where c-chart is definitely needed. Some of the representative types of defects to which c-chart can applied with advantages are:

1. c is number of imperfections observed in a bale of cloth.
2. c is the number of surface defects observed in (i) roll of coated paper or a sheet of photographic film and (ii) a galvanised sheet or a painted, plated or enamelled surface of given area.
3. c is the number of defects of all types observed in aircraft sub-assemblies or final assembly.
4. c is the number of breakdowns at weak sports in insulation in a given length of insulated wire subject to a specified test voltage.
5. c is the number of defects observed in stains or blemishes on a surface.
6. c is the number of soiled packages in a given consignment.
7. c-chart has been applied to sampling acceptance procedures based on number of defects per unit, example, in case of inspection of fairly complex assembled units such as T.V. sets, aircraft engines, tanks, machine-guns, etc., in which there are very many opportunities for the occurrence of defects of various types and the total number of defects of all types found by inspection is recorded for each unit.
8. c-chart technique can be used with advantages in various fields other than industrial quality control, example, it has been applied (i) to accident statistics (both of industrial accidents and highway accidents), (ii) in chemical laboratories and (iii) in epidemiology.

Unit III

Acceptance Sampling

3.1. Introduction

Acceptance sampling is the process of evaluating a portion of the product/material in a lot for the purpose of accepting or rejecting the lot as either conforming or not conforming to quality specification.

Inspection for acceptance purpose is carried out at many stages in manufacturing. There are generally two ways in which inspection is carried out: (i) 100% inspection (ii) sampling inspection

In 100% inspection all the parts or products are subjected to inspection, whereas in sampling inspection only a sample is drawn from the lot and inspected.

A sample may be defined as the number of times drawn from a lot, batch or population for inspection purpose.

3.2. Sampling Inspection

Sampling inspection can be defined as a technique to determine the acceptance or rejection of a lot or population on the basis of number of defective parts found in a random sample drawn from the lot. If the number of defective items does not exceed a predefined level, the lot is accepted, otherwise it is rejected.

Sampling inspection is not a new concept. In our daily life we use sampling inspection in selecting certain consumable items. For example, while purchasing our annual or monthly requirements of wheat, rice or such other food grains we naturally take a handful of grains to judge its quality for taking purchasing decision. If we are not satisfied we take another sample and after two or three samples from the same or different sources we take purchasing decision. Let us take another example, suppose we want to purchase mangoes we normally take one or two mangoes from the lot and taste its quality, if the samples taken are found good we decide to purchase the required quantity.

Similarly, in engineering sampling inspection is preferred because it is more practical, quick and economical as compared to 100% inspection. The main purpose of acceptance sampling is to distinguish between good lots and bad lots, and to classify the lots according to their acceptability or non-acceptability.

Advantages of Sampling Inspection

The advantages of sampling inspection are as follows:

1. The items which are subjected to destructive test must be inspected by sampling inspection only.
2. The cost and time required for sampling inspection is quite less as compared to 100% inspection.
3. Problem inspection fatigue which occurs in 100% inspection is eliminated.
4. Smaller inspection staff is necessary.
5. Less damage to products because only few items are subjected to handling during inspection.
6. The problem of monotony and inspector error introduced by 100% inspection is minimised.
7. The most important advantage of sampling inspection is that, it exerts more effective pressure on quality improvement. Since the rejection of entire lot on the basis of sampling brings much stronger pressure on quality improvement than the rejection of individual articles.

Limitations of Sampling Inspection

1. Risk of making wrong decisions

   However, in sampling inspection, since only a part is inspected, it is inevitable that the sample may not always represent the exact picture obtaining in the lot and hence, there will be likelihood or risk of making wrong decisions about the lot. This wrong decision can be made in two ways. Firstly, a really good lot (that is, containing less proportion of defective than specified) may be rejected because the sample drawn may be bad. Secondly, a really bad lot (that is, a lot containing greater proportion of defectives than specified) may be accepted because the sample drawn may be good. In the former case, the producer has to suffer a risk of his good lots being rejected and hence the associated risk (chance) is called as the producer’s risk. In the letter case, the consumer runs the risk of accepting bad lots and hence the associated risk is called as consumer’s risk.
2. The sample usually provides less information about the product than 100 per cent inspection.
3. Some extra planning and documentation is necessary.

However, in scientific sampling plans, these risks are quantified and the sampling criteria are adjusted to balance these risks, in the light of the economic factors involved.

The success of a sampling scheme depends upon the following factors:

(i) Randomness of samples
(ii) Sample size
(iii) Quality characteristic to be tested

(iv) Acceptance criteria
(v) Lot size

Industrial Uses of Acceptance sampling

1. To determine the quality and acceptability of incoming raw materials, component parts, products etc.
2. To decide the acceptability of semi finished products for further processing as it undergoes the operations from machine to machine or section to section within the factory.
3. To determine the quality of outgoing products.
4. For improving maintaining and controlling the quality of the products manufactured.

3.3. Basic in Acceptance Sampling

In any production process, the producer gets his lot checked at various stages or the customer is anxious to satisfy himself about the quality of goods. An ideal way of doing this, seems to inspect each and every item presented for acceptance. 100 percent inspection should be taken under the following circumstance:

1. The occurrence of a defect may cause less of life or serious causality.
2. A defect may cause serious malfunction of equipments.
3. If testing is destructive like crackers, bulbs, shells etc., it is absolutely nonsensical to talk of hundred percent inspection.

From practical and economic considerations, sampling procedures are adopted such as a lot is accepted or rejected on the basic of samples drawn at random from the lot.

It is noted that, if a scientifically designed sampling inspection plan is used it provides sufficient protection to both producer and consumer. The main objective of infection is to control the quality of product. Sampling inspection ensures that, the quality lot is accepted according to the specifications of the consumer.

The guide lines of a sampling procedure are given below

1. It should give a definite assurance against a passing unsatisfactory lot.
2. The inspection expenses should below as possible.

1. Acceptance Quality Level (AQL)

If a lot has small fraction defective, we do not wish to reject and considered as a good lot. Let p_1 denotes a lot of quality. For small fraction defective,

$$P_r \text{ (Rejecting a lot of quality } p_1) = 0.05$$

This implies that,

$$P \text{ (Accepting a lot of quality } p_1) = 0.95$$

Here, p_1 is known as acceptance quality level and the quality of this lot is satisfactory by the consumers.

2. **Lot Tolerance Percent Defective (LTPD) or Rejecting Quality Level (RQL)**

The consumer is not willing to accept the lot having proportion defective p_1 greater. $100 \times p_1$ is called lot tolerance percent defective. This is the quality level in which consumer regards as rejectable and it is usually named as rejecting quality level.

A lot of quality p_1 is to be accepted at some arbitrary and small fraction of time, usually 10 percent.

3. **Process Average Fraction Defective $(\bar{p})$**

In any production process, the quality of a product tends to settle down to some desired level which may be expected to be more or less the same for every day for a particular machine. If this level could be maintained and if the process is working free from assignable causes of variation, the inspection could often be dispensed with. But in practice as a result of failure of machine and laziness of workers, the quality of the product may suddenly decrease.

The process average of any manufactured product is obtained by computing the percentage of defectives in the product over a long time and it is denoted by $\bar{p}$.

4. **Producer's Risk (α)**

The producer has to face the situation that some good lots will be rejected. The producer might demand sufficient protection against such contingencies happening frequently just as the consumer can claim reasonable protection against accepting too many bad lots. The probability of rejecting a lot with $100\,\bar{p}$ as the process average percent defective is called producers risk (P_p). This probability is denoted by α.

$$\alpha = (P_p) = P \text{ (Rejecting a lot of quality } \bar{p} \text{)}$$

This is also known as Type I error.

5. **Consumer's Risk (β)**

In some critical situations consumer has to face the problem that he may accept certain percentage of undesirable bad lots, which have lots of quality p_1 or greater fraction defective. The probability of accepting a lot with fraction defective p_1 is denoted by β. Usually consumer's risk is taken at 10% level.

$$Pc = P \text{ (Accepting a lot of quality } p_1 \text{)} = \beta$$

This is also known as Type II error.

3.4. Rectifying Inspection Plans

The inspection of the rejected lots and replacing the defective pieces by standard pieces. This process eliminates the number of defectives in the lot and hence to improve the quality of the product. These plans are called rectifying inspection plan and it was first introduced by Dodge and Roming before World War II. This plans enable the manufacture to have an idea about the average quality of the product. The rectification process is performed in different stages of production through the combination of production, the sampling inspection and rectification of rejected lots. Most of the rectifying inspection plans for lot by lot sampling call for 100% inspection of the rejected lots and replacing the defective pieces by good once.

The two important quality control concepts related to rectifying control concepts related to rectifying inspection plans. They are Average Outgoing Quality (AOQ) and Average Total Inspection (ATI).

(i) Average quality of the product after sampling and 100% inspection of rejected lots is called AOQ.

(ii) The average amount of inspection required for the rectifying inspection plan is called ATI.

3.5. Average Outgoing Quality Limit (AOQL)

Sometimes the consumer is guaranteed a certain quality level after inspection - regardless of what quality level is being maintained by the producer. Let the producer's fraction defective, i.e., lot quality before inspection be 'p'. This is termed as 'incoming quality'. The fraction defective of the lot after inspection is known as 'outgoing quality' of the lot. The expected fraction defective remaining in the lot after the application of the sampling inspection plans is termed as AOQ $\tilde{p}$. Obviously, it is a function of the incoming quality 'p'.

Remark

For rectifying inspection single sampling plan calling for 100% inspection of the rejected lots, the AOQ values are given by the formula:

$$\tilde{p} = AOQ = \frac{p(N-n)P_a}{N} \qquad (1)$$

Where N is lot size, n is sample size and P_a is the probability of acceptance of the lot.

If n is small compared with N, then a good approximation of the outgoing quality is given by,

$$\tilde{p} = AOQ = p\,P_a \qquad (2)$$

If the defective pieces found are not repaired or replaced, then the formula must be modified to

$$AOQ = \frac{p(N-n)P_a}{N - np - p(1-P_a)(N-n)} = \frac{p(N-n)P_a}{N - p[nP_a + N(1-P_a)]} \qquad (3)$$

This formula is not generally used and if p is small, there is not much difference between equation (2) and (3).

In general, if p is the incoming quality and a rectifying inspection plan calling for 100% inspection of the rejected lots is used, then the AOQ of the lot will be given by,

$$AOQ = p\,P_a(p) + 0 \times [1 - P_a(p)] = p\,P_a(p) \qquad (4)$$

Because

(i) $P_a(p)$ is the probability of accepting the lot of quality 'p' and when the lot is accepted on the basis of the inspection plan, the outgoing quality of the lot will be approximately same as the incoming lot quality 'p'.

(ii) $1 - P_a(p)$ is the probability of rejection of the lot is rejected after sampling inspection and is subjected to 100% screening and rectification, the AOQ is zero.

For a given sampling plan, the value of AOQ can be plotted for different values of p to obtain the AOQ curve.

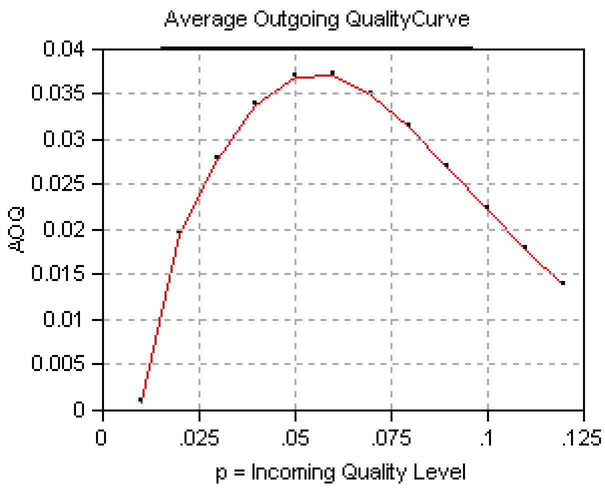

Figure: Average Outgoing Quality Curve

3.6. Average Sample Number (ASN) and Average Total Inspection (ATI)

The average sample number (ASN) is the expected value of the size required to take a decision about the acceptance or rejection of the lot. Suppose the lot is accepted, ASN is equal to the sample size. It is also noted that ASN is a function of incoming lot quality p.

The expected number of items inspected per lot is to arrive a decision as acceptance or rejection or rectification after 100% inspection is called ATI.

ATI is the function of ASN and average size of inspection of remaining units in the lot.

ATI = ASN + (Average size of inspection in the remaining units) (1)

If the lot is accepted on the basis of sampling inspection plan, ATI = ASN. If the lot is rejected, ATI > ASN.

For example, if a single sampling plan number of inspected item in the lot is equal to sample size, if the lot is accepted. i.e. ASN = n. In a single sampling plan after inspecting the sample, the lot is submitted for 100% inspection, the inspected items vary from lot to lot. As we already told, if the lot is equal to sample size (n). If the lot is rejected, the number of inspected items is lot size (N).

Let P_a be the probability of acceptance and 1-P_a be the probability of rejection. As per the above statements of acceptance and rejection, ATI is defined as,

i.e. Sample size and rejection of the lot.

$$ATI = nP_a + N(1 - P_a)$$

$$= nP_a + (N + n - n)(1 - P_a)$$

$$= nP_a + P_a(N - n)(1 - P_a) + n(1 - P_a)$$

$$ATI = n = (N - n)(1 - P_a)$$

As in the case of OC curves, fixing the fraction defectives and find the probabilities of acceptance. Then calculate the values of ATI by using the above relation by taking the fraction defective along x axis and ATI along y axis and get the required ATI curve. The shape of the ATI curve is the mirror image of OC curve.

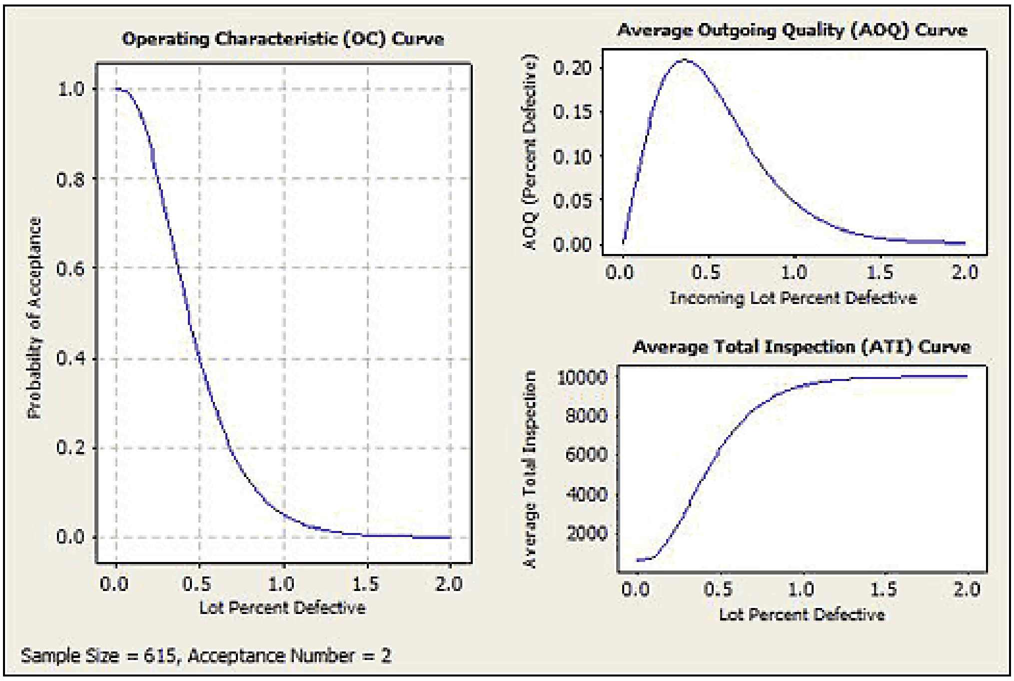

Figure: Operating Characteristic Curve, Average Outgoing Quality Curve and Average Total Inspection Curves

Exercises

1. What is meant by sampling inspection by attributes?
2. What is an acceptance sampling?
3. Define producer's and consumer's risks on operating characteristic curve.
4. Discuss the following:

(i) AOQ and AOQL (ii) Types of OC curves (iii) AQL and LTPD

Unit IV

Acceptance Sampling by Attributes

4.1. Introduction

In attributes sampling, a predetermined number of units from each lot is inspected. Each unit is graded as conforming or nonconforming. A nonconforming unit is defined as a unit that does not meet product specifications for one or more quality characteristics. If the number of nonconforming units is less than the prescribed minimum, the lot is accepted; otherwise, the lot is rejected. There are several types of plans for attributes sampling. Four of these are single sampling, double sampling, multiple sampling and sequential sampling plans. These are discussed in the following sections.

4.2. Single Sampling Plan

In any production process 100% inspection requires more time, more inspection cost and soon. For reducing time and cost, we may apply sampling procedure. Dodge and Roming have established single sampling plan, double sampling plan, multiple sampling plan, etc.

The working principle of single sampling plan is described as follows:

A Single Sampling Plan is denoted as $\begin{pmatrix} N \\ n \\ c \end{pmatrix}$. Let N be the lot size, n be the sample size and c be the acceptance number. The acceptance number is otherwise known as maximum allowable number of defectives in the sample.

Select a random sample of size n from a lot of size N.

1. Inspect all the articles in the sample.
2. The number of defectives in the sample be ‘d’.
3. If $d \leq c$, accept the lot, replacing defective pieces found in the sample by non-defective (standard) pieces.
4. If $d > c$, reject the lot and we inspect the entire lot and replace all the defective pieces by standard pieces.

The working principle of Single Sampling Plan is exhibited in the following chart:

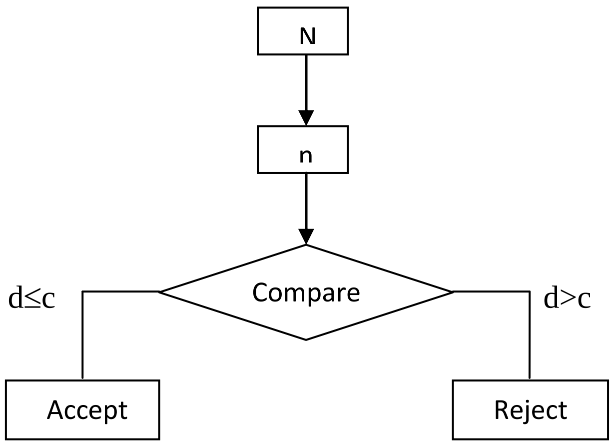

Figure: Single Sampling Plan

For analysing the quality of the product, quality control engineers are interested to design Operating Characteristic (OC) curve, Average Outgoing Quality (AOQ) curve, Average Sample Number (ASN) curve and Average Total Inspection (ATI) curve.

4.3. OC, AOQ, ATI and ASN for Single Sampling Plan

In acceptance sampling plan, there are many curves to detect the process variation and find out probability of acceptance, probability of rejection, ASN and so on. Among the curves, OC curve is a simple and basis of either fraction defective p or number of defectives in sample (np) verses probability of acceptance.

One may able to construct OC curve for single sampling plan with finite or infinite size. OC curve is classified into two types such as type A and type B OC curves. The construction of OC curves for single sampling plan is described as follows:

Consider a single sampling plan $\begin{pmatrix} N \\ n \\ c \end{pmatrix}$. Let p be the fraction defective gives Np is the defective in lot and x is the number of defectives in sample.

According to the principle of hypergeometric distribution, probability of x defectives with fraction defective p is given by

$$g(x,p) = \frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}} \qquad (1)$$

Probability of acceptance of the given lot is defined as the cumulative probabilities of x = 0, 1, …, c defectives. By using equation (1) we have

$$L(p) = P_a = \sum_{x=0}^{c} g(x,p) = \sum_{x=0}^{c} \frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}} \qquad (2)$$

Suppose p is very small, the RHS of equation (2) it is approximately equal to binomial expression of $\left[\left(1-\frac{n}{N}\right)+\frac{n}{N}\right]^{Np}$. When apply binomial property to the equation (2), it reduces to

$$L(p) = \sum_{x=0}^{c} \binom{Np}{x}\left(\frac{n}{N}\right)^{x}\left(1-\frac{n}{N}\right)^{Np-x} \qquad (3)$$

In equation (3), we considered $\frac{n}{N}$ is very small and Np is large ($Np \to \infty$). Then the equation (3) becomes

$$L(p) = \sum_{x=0}^{c} e^{-np}\frac{(np)^{x}}{x!} \qquad (4)$$

By using different values of any p and c, we get different cumulative probabilities from cumulative Poisson distribution tables. On taking 'np' along x axis and L(p) along y axis and draw the required OC curve.

Determine the parameters of Single Sampling Plan: (Determine n and C)

Consider a single sampling plan $\begin{pmatrix} N \\ n \\ c \end{pmatrix}$. Here N be the lot size, n be the sample size and c be the acceptance number. In any single sampling plan the lot size N either finite or infinite and always assume that it is a known quantity. But n and c are generally unknown quantities and they are to be determined.

Let us assume p be the fraction defective or a lot of incoming quality. Out of N units, the number of defective pieces is taken as 'Np' and number of non-defective pieces is 'N-Np'. Suppose x is the number of defectives in the sample then find out the probability distribution function of x defectives in the lot. The number of ways of selecting x defectives from Np defectives is $\binom{Np}{x}$. Similarly the number of ways of selecting (n-x) non-defectives from, the number of non-defectives in the lot is $\binom{N-Np}{n-x}$.

The number of ways of selecting n units from N units is $\binom{N}{n}$.

From the above statement, the probability of getting x defectives is given by

$$g(x,p)=\frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}} \tag{1}$$

This is the probability mass function of hypergeometric distribution.

The probability of accepting a lot with fraction defective p is obtained by taking the summation x=0, 1, …, c in equation (1) and get

$$P_a=\sum_{x=0}^{c} g(x,p)=\sum_{x=0}^{c}\frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}} \tag{2}$$

This equation (2) is also known as consumer's risk.

$$\text{i.e. } P_c=\sum_{x=0}^{c} g(x,p)=\sum_{x=0}^{c}\frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}} \tag{3}$$

On the other hand producer's risk is given by

$$P_p=1-P_c=1-\sum_{x=0}^{c}\frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}} \tag{4}$$

Equations (3) and (4) are enough to find the values of n and c, but it is cumbersome to solve the equations. So we reduce equation (3) and (4) in the form of the probability mass function of binomial distribution.

When p is very small the right hand side of equation (3) is approximately equal to the first c+1 terms of binomial expansion $\left[\left(1-\frac{n}{N}\right)+\frac{n}{N}\right]^{Np}$

Then equation (3) becomes

$$P_c=\sum\binom{Np}{x}\left(\frac{n}{N}\right)^{x}\left(1-\frac{n}{N}\right)^{Np-x} \tag{5}$$

In this stage we considered $\frac{n}{N}$ is very small, Np is very large and 'np' is taken as Poisson variate and also taking the limit as $Np\to\infty$, then equation (5) reduces to

$$P_c = \sum_{x=0}^{c} e^{-np} \frac{(np)^x}{x!} \qquad (6)$$

Similarly equation (4) reduces to

$$P_p = 1 - P_c = 1 - \sum_{x=0}^{c} e^{-np} \frac{(np)^x}{x!} \qquad (7)$$

On solving equation (6) and (7) we get a required values of the parameter n and c.

4.4. Double Sampling Plan

Dodge and Roming have proposed another sampling scheme which is called second sampling method or double sampling procedure. In this method, a second sampling permitted if the first samples fails. If we are not able to take decision based on first sample, then draw second sample and conclude decision on the basis of second sample. The double sampling plan is denoted as $\begin{pmatrix} N \\ n_1 \\ c_1 \\ n_2 \\ c_2 \end{pmatrix}$. Here N is the lot size, n_1 is the first sample size. c_1 is the acceptance number for first sample or maximum allowable number of defectives in the first sample. n_2 is the second sample size. c_2 is the acceptance number for both samples combined. d_1 is the number of defective items in the first sample, d_2 is the number of defective in the second sample.

The working principle of double sampling plan is described as follows

1. Consider a lot of size N from a production plan.
2. Take a sample of size n_1 from the lot.
3. Inspect the first sample and find out the number of defective items, let it be 'd_1'.
4. If $d_1 \le c_1$, accept the lot and replace the bad items by standard items.
5. If $d_1 > c_2$, reject the lot and apply 100% inspection in the lot. Remove all bad items and include standard ones to adjust the lot size as N.
6. If $c_1 + 1 < d_1 \le c_2$ then take a second sample of size n_2 from the remaining lot.
7. If $d_1 + d_2 \le c_2$ accept the lot and replace the bad items by standard items.
8. If $d_1 + d_2 > c_2$ reject the whole lot and apply 100% inspection.

Replace defective items by standard items. This working principle of double sampling plan is exhibited in the following chart:

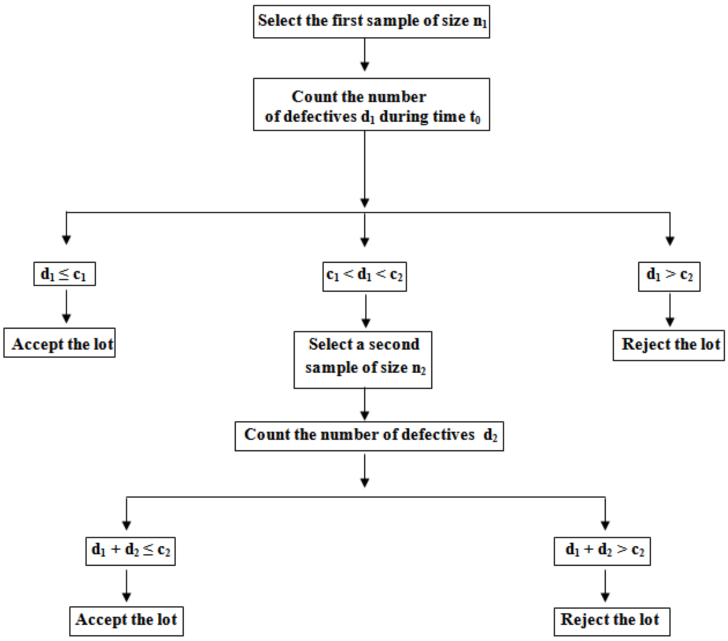

Figure: Double Sampling Plan

4.5. OC, ASN and ATI for Double Sampling Plan

Consider a double sampling plan $\begin{pmatrix} N \\ n_1 \\ c_1 \\ n_2 \\ c_2 \end{pmatrix}$. Here N is the lot size. n_1 *and* n_2 are fixed first and second sample sizes respectively, c_1 *and* c_2 are acceptance numbers corresponding to first and both combined samples respectively. The construction of OC curve for double sampling plan is very cumbersome, since the selection of second sample depends upon the number of nature of first sample. Let x be the number of defectives in the first sample, if x lies between (0, c_1), then we accept the lot and find the probability of acceptance. This implies that there is no chance to select second sample. We know that the probability of getting x defective with fraction defective p is g(x, p). The number of defectives 'x' follows hypergeometric distribution and its probability mass function is given by,

$$g(x,p)=\frac{\binom{Np}{x}\binom{N-Np}{n_1-x}}{\binom{N}{n_1}} \qquad (1)$$

The cumulative probability of x defectives (x = 0,1,2,…,c_1) is called probability of acceptance of first sample. Summing the equation (1), by apply x = 0,1,2,…,c_1 and get the probability of acceptance of first sample.

$$P_{a1}=\sum_{x=0}^{c_1} g(x,p)$$

$$= \sum_{x=0}^{c_1} \frac{\binom{Np}{x}\binom{N-Np}{n_1-x}}{\binom{N}{n_1}} \qquad (2)$$

Suppose number of defectives (x) in the first sample satisfy the inequality $c_1 + 1 \le x \le c_2$ be selected second sample of size n_2. After inspecting the second sample and observed that there are 'y' defectives. In this stage, we have to construct the conditional probability of 'y' defectives in the second sample when there are x defectives in the first sample. Let it be h(y, p/x). The number of ways of selecting y defectives from Np-x defectives is $\binom{Np-x}{y}$.

In the second sample, there are $n_2 - y$ non-defectives and $N - n_1 - (Np - x)$ non-defectives are available in the lot. The number of ways of selecting $n_2 - y$ non-defectives from $[N - n_1 - (Np - x)]$ non-defectives is $\binom{(N-n_1)-(Np-x)}{n_2-y}$.

Finally the sample of size n_2 is selected from the remaining lot $N - n_1$. In this case, the number of ways selecting n_2 units is $\binom{N-n_1}{n_2}$. By using the above statements, the conditional probability of 'y' defectives in the second sample when there are x defectives in the first sample is obtained as

$$h(y, p/x) = \frac{\binom{Np-x}{y}\binom{N-n_1-(Np-x)}{n_2-y}}{\binom{N-n_1}{n_2}} \qquad (3)$$

Let P_{a2} be the probability of acceptance in the second sample under the condition that 'x' defectives in the first sample and y defectives in the second sample based on the non-acceptance if the first sample with x defectives,

$$P_{a2} = \sum\sum g(x, p) \times h(y, p/x) \qquad (4)$$

By using the equations (1), (2) in equation (4),

We get

$$P_{a2} = \sum_{y=0}^{c_2-x} \sum_{x=c_1+1}^{c_2} \frac{\binom{Np-x}{x}\binom{N-Np}{n_1-x}}{\binom{N}{n_1}} \frac{\binom{Np-x}{y}\binom{N-n_1-(Np-x}{n_2-y}}{\binom{N-n_1}{n_2}} \qquad (5)$$

By adding equation (2) and (5), we get the probability of acceptance P_a in double sampling plan as

$$P_a = P_{a1}+P_{a2} \quad (6)$$

For different values of lot quantity (fraction defective) ‘p’ we compute corresponding probabilities P_a and by considering p and P_a, we can draw the OC curve for the given double sampling plan.

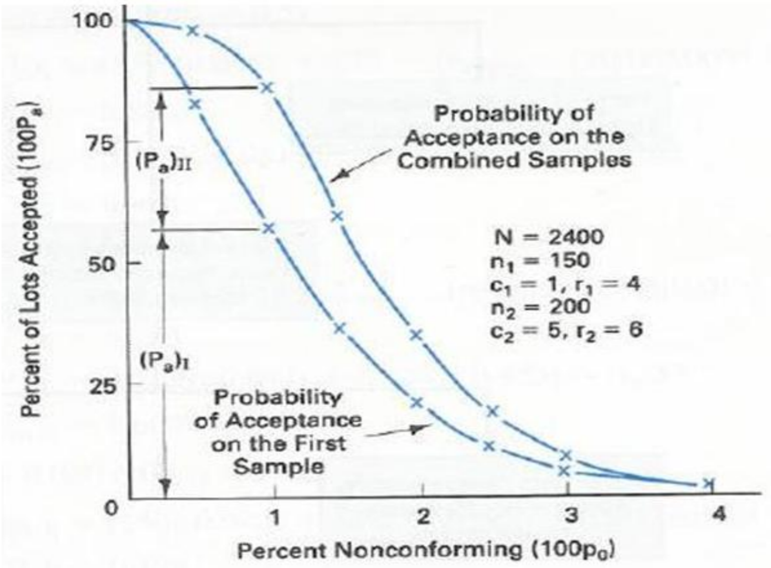

Figure: OC Curve for Double Sampling Plan

4.6. Comparation of Single Sampling and Double Sampling Plans

1. Single sampling plan are simple, easy to design and administer.
2. An advantage of double sampling plan over single sampling plan seems to be psychological.
3. Leyman can feel that reject a lot on the basis of single sampling plan is undesirable but they are satisfied when the lot is rejecting after inspecting the second sample.
4. Double sampling plan involves on the average less amount of inspection than that of single sampling plan for the same quality assurance.
5. Under double sampling plan, good quality lot will be accepted and bad quality lot will be rejected on the basis of first sample.
6. When a lot is rejected on the basis of second sample, without completely inspecting the entire lot, double sampling plan requires 25% or 33% less inspection than single sampling plan.
7. Unit cost of inspection for double sampling plan may be higher than that for single sampling plan.
8. OC curves for double sampling plan are steeper than OC curves for single sampling plan. This implies that the discriminant power of double sampling plan is higher than that of single sampling plan.

Also, we compare SSP and DSP in the following ways:

	Single Sampling Plan	Double Sampling plan
1. Average number of pieces inspected per lot	Largest	In between single and multiple plans
2. Cost of administration	Lowest	In between single and multiple plans
3. Information available regarding prevailing quality level	Largest	In between single and multiple plans
4. Acceptability to producers	Less (gives only one change of passing the lot)	Most acceptable

Types of OC curves

Dodge and Roming have classified the OC curves into two types such as type A and type B OC curves. Type A OC curve are drawn for the sampling plans with finite lots. The curves give the probability of acceptance of the lot quality. These curves should be drawn by computing probability of acceptance based on hypergeometric distribution or binomial distribution or Poisson distribution. Since the above discrete distributions, they give discontinuous curves. But in practice it is drawn as continuous curves.

Type B OC curves are drawn for the sampling plans which are having infinite lot size. The curves gives the probability of acceptance of product quality. Type B curves are drawn based on the probability of acceptance relative to binomial distribution. The probabilities from binomial distribution give exact OC curve. But the probabilities from Poisson distribution give satisfactory approximation curve.

In general, if the sample size n is not more than one-tenth of the lot size N, type A and type B OC curve may be considered as identical. Type A OC curve may be desirable in evaluating consumer’s risk with respect to individual lots. Procedures risk is approximately measured based on type B OC curve.

Type A curve always falls below the type B curve, it follows that the use of type B curves tends to give consumer’s risk. That is very high.

Average Outgoing Quality (AOQ) for Single Sampling Plan

Consider a single sampling plan $\begin{pmatrix} N \\ n \\ c \end{pmatrix}$. Average quality of the product after sampling inspection is called AOQ. Take a sample of size n from the lot of size N. Let p be a fraction defective and P_a be the probability of acceptance of the given lot. Similarly $1\text{-}P_a$ is the probability of rejection of the given lot. The number of defective units in the sample is consider as ‘np’.

After drawing the sample, the remaining units in the lot is N-n. It implies that number of defective units in the remaining lot is (N-n)p out of (N-n) units.

The average of acceptable units of AOQ

$$AOQ = \frac{k(N-n)pP_a}{kN}$$

$$AOQ = \frac{(N-n)pP_a}{N} \text{ If N is finite.}$$

If N is very large $N \to \infty$,

$$AOQ = \frac{(N-n)pP_a}{N}$$

$$= \left(\frac{N}{N} - \frac{n}{N}\right)pP_a$$

$$= \left(1 - \frac{n}{N}\right)pP_a$$

$$AOQ = pP_a$$

If N is infinite. The maximum value of AOQ is called Average Quality Level or maximum available outgoing quality. By taking 'p' along x axis and 'AOQ' along y axis, we get required AOQ curve.

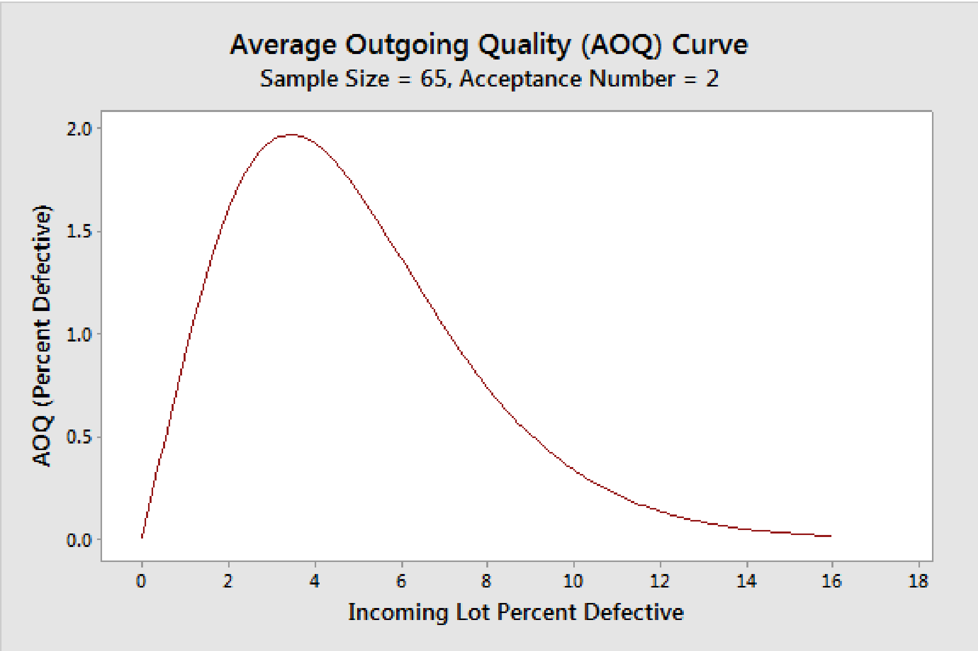

Figure: Average Outgoing Quality for Single Sampling Plan

Average Sample Number (ASN) and Average Amount of Total Inspection (ATI)

The ASN is the expected value of the size required to take a decision about the acceptance or rejection of the lot. Suppose the lot is accepted, ASN is equal to the sample size. It is also noted that ASN is a function of incoming lot quality p.

The expected number of items inspected per lot is to arrive a decision as acceptance or rejection or rectification after 100% inspection is called ATI.

ATI is the function of ASN and average size of inspection of remaining units in the lot.

ATI = ASN + (Average size of inspection in the remaining units).

If the lot is accepted on the basis of sampling inspection plan, ATI = ASN. If the lot is rejected, ATI > ASN.

In single sampling plan, number of inspected item in the lot is equal to sample size, if the lot is accepted. i.e. ASN = n. In a single sampling plan after inspecting the sample, the lot is submitted for 100% inspection, the inspected items vary from lot to lot. As we already told, if the lot is equal to sample size (n). If the lot is rejected, the number of inspected items is lot size (N).

Let P_a be the probability of acceptance and $1-P_a$ be the probability of rejection. As per the above statements of acceptance and rejection, ATI is defined as,

i.e. Sample size and rejection of the lot.

$$ATI = nP_a + N(1-P_a)$$

$$= nP_a + (N+n-n)(1-P_a)$$

$$= nP_a + P_a(N-n)(1-P_a) + n(1-P_a)$$

$$ATI = n + (N-n)(1-P_a)$$

As in the case of OC curves, fixing the fraction defectives and find the probabilities of acceptance. Then calculate the values of ATI by using the above relation by taking the fraction defective along x axis and ATI along y axis and get the required ATI curve. The shape of the ATI curve is the mirror image of OC curve.

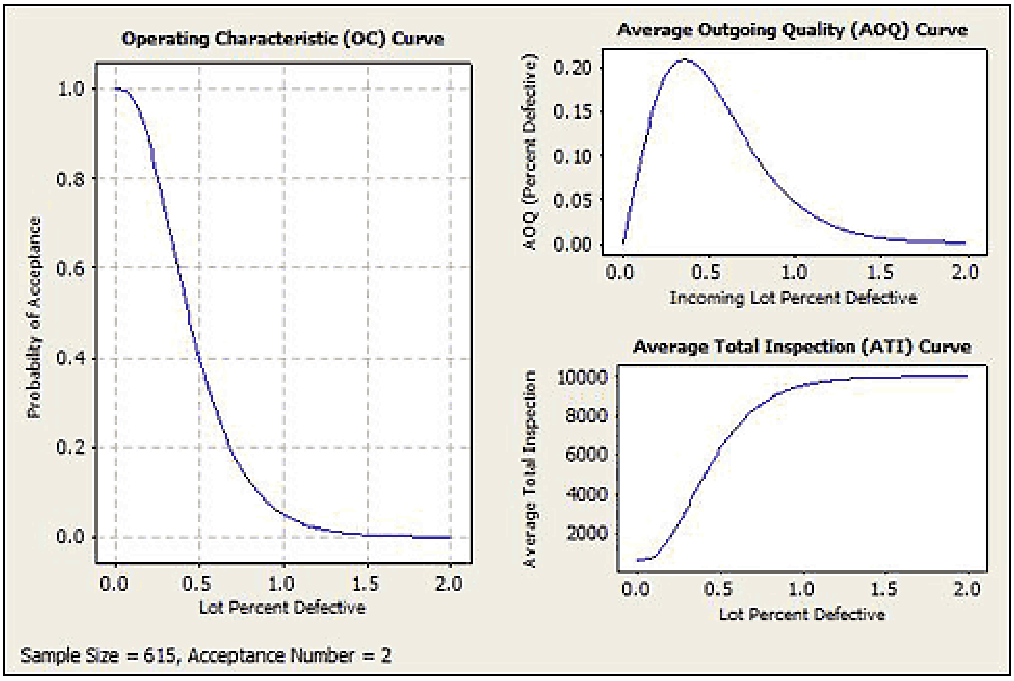

Figure: Operating Characteristic Curve, Average Outgoing Quality Curve and Average Total Inspection Curves for Single Sampling Plan

ASN curve for Double Sampling Plan

Consider a double sampling plan with usual notations as $\begin{pmatrix} N \\ n_1 \\ c_1 \\ n_2 \\ c_2 \end{pmatrix}$. On the basis of inspecting the number of units in any sampling plan, we have to construct the functions of ASN and ATI curves. From a double sampling plan, we select a sample of size n_1 and inspect the units. After inspecting the units we have to take a decision either accept or reject the lot. Let p_1 be the probability of taking a decision. In the first stage sample number is considered as $n_1 p_1$. Again we draw second sample of size n_2 and inspect the units. In this stage, there are n_1 *and* n_2 units are inspected and the corresponding probability of not taking any decision is $(n_1 + n_2)(1 - p_1)$.

By adding the above two sample sizes and get the required ASN.

$$\therefore ASN = n_1 p_1 + (n_1 + n_2)(1 - p_1)$$

$$= n_1 p_1 + n_1(1 - p_1) + n_2(1 - p_1)$$

$$ASN = n_1 + n_2(1 - p_1)$$

By taking the values of p along x axis and ASN along y axis, we get ASN curve.

ATI curve for Double Sampling Plan

Consider a double sampling plan with usual notations as $\begin{pmatrix} N \\ n_1 \\ c_1 \\ n_2 \\ c_2 \end{pmatrix}$. We select a sample of size n_1 and inspect the units. Let P_{a1} be the probability of acceptance of the lot on the basis of first sample. Here, the number of inspected items is taken as

$$n_1 P_{a1} \tag{1}$$

Suppose the lot is not accepted on the basis of first sample, we draw second sample of size n_2. Let P_{a2} be the probability of acceptance on basis of second sample when the first sample is already inspected. In the second stage, the number of inspected items for both samples is considered as

$$(n_1 + n_2)P_{a2} \tag{2}$$

Suppose the lot is rejected after the second sampling is inspected, we apply 100% inspection for the whole lot of the size N. In this stage, the probability of rejection is taken as $1-P_a$. In the rejection level, number of inspected item is

$$N(1 - P_a) \tag{3}$$

By adding the above stated inspected items, we get required expression for ATI curve.

$$\therefore ATI = n_1 P_{a1} + (n_1 + n_2)P_{a2} + N(1 - P_a)$$

$$= n_1 P_{a1} + (n_1 + n_2)(P_a - P_{a1}) + N(1 - P_a)$$

$$= n_1 P_{a1} + (n_1 + n_2)[(1 - P_{a1}) - (1 - P_a)] + N(1 - P_a)$$

$$ATI = n_1 + n_2(1 - P_{a1}) + [N - (n_1 + n_2)](1 - P_a)$$

For different values of lot quality, we have to calculate probability of acceptance for first sample, combined sample and overall probabilities of acceptance P_a, by using lot quality p and the corresponding probabilities of acceptance P_a, we draw we required ATI curve which shows the mirror image of OC curve.

Problem 1:

A single sampling plan uses a sample size of 15 and an acceptance number 1. Using hypergeometric probabilities, compute the probability of acceptance of lots of 50 articles 2% defective.

Solution:

Given

N=50, n=15, c=1, p'=0.02 (2%)

Number of defective articles = 0.02*50=1

Number of non-defective article = 50-1=49

$$P_0 = \frac{49C_{15}}{50C_{15}}$$

$$= \frac{49!}{34!\times 15!}\times\frac{35!\times 15!}{50!} = 0.7$$

$$P_1 = \frac{1C_1 \times 49C_{14}}{50C_{15}}$$

$$= \frac{49!}{35!\times 14!}\times\frac{35!\times 15!}{50!} = 0.3$$

Probability of acceptance = 0.7+0.3=1.

Problem 2:

The lot size N is 2000 in a certain AOQL inspection procedure. The desired AOQL of 2% can be obtained with any one of the three sampling plans. These are:

(i) N=65, c=2
(ii) N=41, c=1
(iii) N=18, c=0

If a large number of lots 0.3%defective are submitted for acceptance, what will be the average number of units inspected per lot under each of these sampling plans?

Solution:

(i) Now

$$np' = \frac{65\times 0.3}{100} = 0.195$$

From table G, for $np' = 0.195$ and c=2, $P_a = 0.999$

Average number of items inspected per lot $= P_a.n + (1 - P_a)N$

= 0.999*65+0.001*2000=66.935 (say 67)

(ii) $np' = \frac{41\times 0.3}{100} = 0.123$

From table G, for $np' = 0.123$ and c=1, $P_a = 0.993$

Average number of items inspected per lot

=0.993*41+0.007*2000=54.959 (say 55)

(iii) $np' = \frac{18 \times 0.3}{100} = 0.054$ for $np' = 0.054$ and c=0, $P_a = 0.947$

Average number of items inspected per lot

=0.947*18+0.053*2000=123.046 (say 124)

Problem 3:

(a) Explain the step by step for constructing the OC curve for a single sampling plan.
(b) Draft the OC curve of the single sampling plan: n=300, c=5.

Solution:

(a) The step by step method of constructing the OC curve for a single sampling plan is as follows:

1. Step up table headings and the P_a column as follows:

n	np'	p'	P_a	$P_a p'$
			0.98	
			0.95	
			0.70	
			0.50	
			0.20	
			0.05	
			0.02	

where, n=sample size, np'=number of defectives, p'=fraction defective, P_a=probability of acceptance, $P_a p'$=AOQ=Average Outgoing Quality.

The chosen of P_a will give ordinate values which, when co-ordinated with p' values to be derived, will facilitate construction of an OC curve.

2. Search Table G under the given np' value until the desired P_a (or close value to desired P_a) is located.

   (If the exact value is not found, the value in the P_a column should be changed to correspond with the one selected.
3. Place the np' value associated with the selected P_a in the np' column.
4. Divide the np' value by n. This will give the p' co-ordinate of P_a for the OC curve.

(b) To draft the OC curve for the single sampling plan, n=300, c=5.

1. **Table construction**

n	np'	p'	P_a	$P_a p'$
300			0.98	
300			0.95	
300			0.70	
300			0.50	
300			0.20	
300			0.05	
300			0.02	

2. **Finding np' and p'**

Search through Table, under np' =5 discloses P_a value of 0.983. This is the closest value to 0.98. The np' value associated with a P_a value of 0.983 is 2.0. This value of np', when divided by n=300 gives p' value of 0.0067. The same procedure is followed for each of the other P_a values until the table is computed.

N	np'	p'	P_a	$P_a p'$
300	2.0	0.0067	0.983	0.0065
300	2.6	0.0087	0.951	0.00827
300	4.4	0.0147	0.72	0.0106
300	5.6	0.0187	0.512	0.00957
300	7.8	0.025	0.210	0.00526
300	10.5	0.035	0.05	0.00175
300	12.0	0.02	0.02	0.0008

The $P_a p'$ column is provided to give the necessary values for the graphical of an AOQ curve with p' being the abscissa and $P_a p'$ the ordinate.

Exercises

1. State the advantages and limitation of acceptance sampling over 100% inspection.
2. Explain the OC curve with reference to sampling inspection and the meaning of the term:
   (i) AQL
   (ii) LTPD
   (iii) Producer’s risk
   (iv) Consumer’s risk
3. State difference between Single Sampling plan and Double Sampling plan.
4. What is ATI? How will you compute the ATI for single sampling and double sampling plans?
5. Describe the principle of Single sampling plan.
6. Describe the principle of double sampling plan.
7. Explain the principles of double sampling plan and construct OC, ASN and ATI curves.

Unit - V

Variable Sampling Plan

5.1. Introduction

Acceptance sampling by variable (or) variable sampling plan can be used if a quality characteristic is measured on a numeric scale or continuous scale and is known to follow some statistical distribution. This type of plan is based on sample measurements of the mean and standard deviation.

Advantages

1. For a given quality protection, smaller samples may be used in variable sampling plan than that with attribute sampling plan.
2. Variables information gives a better basis for quality improvements.
3. The conformance and non-conformance are used to estimate quality characteristic.
4. Variable information may give better basis for quality in acceptance decisions.
5. Errors of measurements are disclosed in variable information.
6. The primary advantage of using variable sampling plan is that more information can be obtained about the quality characteristic than when using an attribute sampling plan.

Disadvantages

1. The main disadvantage is that a separate sample plan is needed for each quality characteristic being investigated.
2. In variable sampling plan, for each quality characteristic we have to assume a distribution. In some cases, the assumed distribution and actual distribution are different in nature. This implies that quality protection may occur differently.
3. There is a possibility that a lot may be rejected even if it does not contain any defective items. This may cause some disagreements between customer and business man.

The Normality Assumption

When using acceptance sampling by variables procedures, it is generally assumed that the samples come from a normal population that is, the measured quality characteristic is assumed to be normally distributed. Consequently, a functional relationship exists between the measured quality characteristic and its mean and standard deviation.

5.2. Types of Variable Sampling Plans

There are two types of variable sampling plans. One type is used to control a process parameter and the other type is used to control the lot percent defectives. Variable sampling plans for a process parameter are designed to control the mean or standard deviation of a quality characteristic. Variable sampling plans for lot percent defectives are designed to determine the proportion of product that is in excess of a specified limit.

1. Variable Sampling Plans for a process parameter

(a) Variable Sampling Plans to estimate the process average with a single specification limit when σ known
(b) Variable Sampling Plans to estimate the process average with double specification limits when σ known
(c) Variable Sampling Plans to estimate the process average with a single specification limits when σ unknown
(d) Variable Sampling Plans to estimate the process average with double specification limits when σ unknown

2. Variable Sampling Plans to estimate the lot percent defectives

(a) Variable Sampling Plans with a single specification limit for large sample size when σ unknown
(b) Variable Sampling Plans with a single specification limit for large sample size when σ known
(c) Variable Sampling Plans with a double specification limit for large sample size when σ unknown
(d) Variable Sampling Plans with a double specification limit for large sample size when σ known

5.3. Variable Sampling Plan for a Process Parameter

Variable sampling plans for a process parameter are most likely to be used in sampling products that are submitted in bulk (for example, bags, and boxes). This type of sampling is concerned with the average quality of the product or with the variability in its quality.

(a) Variable Sampling Plans to Estimate the Process Average with a Single Specification Limit – σ Known

To design a single sampling plan for a process average with a single acceptance limit, four characteristics of the plan need to be identified. They are $\overline{X}_1$ (the average value of the quality characteristic for which the probability of acceptance is high), $\overline{X}_2$ (the average value of the quality characteristic for which the probability of acceptance is low), α (the probability of rejecting a lot that meets the specified quality level) and β (the probability of accepting a lot that does not meet the specified quality level). The OC curve of the single sampling plan should pass through two specified points $\left(1-\alpha, \overline{X}_1\right)$ *and* $\left(\beta, \overline{X}_2\right)$. The single sampling plan can then be derived as follows. First, obtain the value of standard normal deviate so that the area under the standard normal curve is (1-α) call this value Z_1. Likewise, obtain the value of the standard normal deviate for β call this Z_2. Let $\bar{X}_a$ be the acceptance limit. By the normality assumption, we have

$$Z_1 = \frac{\bar{X}_1 - \bar{X}_a}{\sigma/\sqrt{n}} \quad (1)$$

$$Z_2 = \frac{\bar{X}_2 - \bar{X}_a}{\sigma/\sqrt{n}} \quad (2)$$

It is assumed, in this case, that the standard deviation σ is known and constant. Notice that the equations have two unknown variables $\bar{X}_a$ and n. Values for these variables can be obtained by solving the two equations (1) and (2) simultaneously. The resulting equations for n and $\bar{X}_a$ are

$$n = \left[\frac{(Z_2 - Z_1)\sigma}{\bar{X}_2 - \bar{X}_1}\right]^2 \quad (3)$$

and

$$\bar{X}_a = \frac{Z_2\,\bar{X}_1 - Z_1\,\bar{X}_2}{Z_2 - Z_1} \quad (4)$$

If the resulting value for n is not an integer, it should be rounded up to the next higher integer value.

Construction of an OC curve

The OC curve of a single sampling plan for a process with a single acceptance limit is constructed by plotting various values of the process average versus the probability of acceptance of each value. The probability of acceptance for a process average $\bar{X}$ under a sampling plan with parameters $\bar{X}_a$ and n is obtained by calculating Z, the standard normal deviate, as

$$Z = \frac{\bar{X} - \bar{X}_a}{\sigma/\sqrt{n}} \quad (5)$$

if an upper acceptance limit is required or

$$Z = \frac{\bar{X}_a - \bar{X}}{\sigma/\sqrt{n}} \quad (6)$$

if a lower acceptance limit is required. The area under the standard normal curve that is less than the value of normal deviate is the probability of acceptance value.

Problem 1:

Suppose that a company wants to set up a variables sampling plan for steel bars based on tensile strength. The company would like to accept steel bars with an average tensile strength of 10000 psi or higher 95% of the time. Conversely, the company would like to reject steel bars with an average tensile strength of 9950 psi or less 90% of time. In other words $\overline{X}_1 = 10000, \overline{X}_2 = 9950, \alpha = 0.05 \textit{ and } \beta = 0.10$. Suppose that the standard deviation is known to be 100 psi.

Solution:

The values of the standard normal deviates for $(1-\alpha)$ and β are 1.645 and -1.282 respectively. Substituting these values into given equations

$$n = \left[\frac{(Z_2 - Z_1)\sigma}{\overline{X}_2 - \overline{X}_1} \right]^2$$

$$n = \left[\frac{(-1.282 - 1.645)\,100}{9950 - 10000} \right]^2 = 34.27$$

and

$$\overline{X}_a = \frac{Z_2\,\overline{X}_1 - Z_1\,\overline{X}_2}{Z_2 - Z_1}$$

$$\overline{X}_a = \frac{-1.282(10000) - 1.645(9950)}{-1.282 - 1.645} = 9972$$

Rounding the value of n to the next higher integer gives n=35. The variables sampling plan is as follows. Take a random sample of 35 steel bars from each lot and compute the average tensile strength. If the average tensile strength of the sample is 9972 psi or higher accept the lot, otherwise reject it.

(b) Variable Sampling Plans to Estimate the Process Average with Double Specification Limits - σ is known

In statistical hypothesis, the sample statistics and population parameters are used to test the validity of the population parameter is not known we must use sample statistic. On the other hand if population parameters are known (given), we use the given parameters.

Variable sampling plan with specified upper limit and lower limit are specified is constructed by using some characteristics. In this case variable sampling plan consist of five characteristics and they are $\alpha, \beta, \overline{X}_1, \overline{X}_{2U} \textit{ and } \overline{X}_{2L}$. Here α is the probability of rejecting a lot that means the specified quality level (producer's risk), β is the probability of accepting a lot that does not meet the specified quality level (consumer's risk). $\overline{X}_{2U} \textit{ and } \overline{X}_{2L}$ are upper and lower values of $\overline{X}_2$ respectively. Let $z_1 \textit{ and } z_2$ be standard normal variates corresponding to

$1-\frac{\alpha}{2} and 1-\frac{\beta}{2}$ respectively. Our aim is to estimate $\overline{X}_U, \overline{X}_L$ and n. We know that the standard error of sample average is $\frac{\sigma}{\sqrt{n}}$.

$$\text{(i.e.) } SE(\overline{X}) = \frac{\sigma}{\sqrt{n}}$$

For estimating the unknown, we have to form four standard normal variables by using specification limits.

$$z_1^{\,U} = \frac{\overline{X}_U - \overline{X}_1}{\sigma/\sqrt{n}} \tag{1}$$

$$z_1^{\,L} = \frac{\overline{X}_1 - \overline{X}_L}{\sigma/\sqrt{n}} \tag{2}$$

$$z_2^{\,U} = \frac{\overline{X}_{2U} - \overline{X}_U}{\sigma/\sqrt{n}} \tag{3}$$

$$z_2^{\,L} = \frac{\overline{X}_L - \overline{X}_{2L}}{\sigma/\sqrt{n}} \tag{4}$$

There are four equations and three unknown variables $\overline{X}_U, \overline{X}_L$ and n. Now subtract equation (2) from equation (3),

$$(3)-(2) \Rightarrow$$

$$z_2^{\,U} - z_1^{\,L} = \frac{1}{\sigma/\sqrt{n}}\left[\overline{X}_{2U} - \overline{X}_U - \{\overline{X}_1 - \overline{X}_2\}\right]$$

$$\left[z_2^{\,U} - z_1^{\,L}\right)\frac{\sigma}{\sqrt{n}} = \overline{X}_{2U} - \overline{X}_U - \overline{X}_1 + \overline{X}_L \tag{5}$$

$$Again \frac{(3)}{(2)} \Rightarrow$$

$$\frac{z_2^{\,U}}{z_2^{\,L}} = \frac{\left(\overline{X}_{2U} - \overline{X}_U\right)/\left(\sigma/\sqrt{n}\right)}{\left(\overline{X}_1 - \overline{X}_L\right)/\left(\sigma/\sqrt{n}\right)}$$

$$\frac{z_2^{\,U}}{z_2^{\,L}} = \frac{\overline{X}_{2U} - \overline{X}_U}{\overline{X}_1 - \overline{X}_L} \tag{6}$$

$$\frac{(4)}{(2)} \Rightarrow \frac{z_2^{\,L}}{z_1^{\,L}} = \frac{\left(\overline{X}_L - \overline{X}_{2L}\right)/\left(\sigma/\sqrt{n}\right)}{\left(\overline{X}_1 - \overline{X}_L\right)/\left(\sigma/\sqrt{n}\right)}$$

$$= \frac{\overline{X}_L - \overline{X}_{2L}}{\overline{X}_1 - \overline{X}_L} \tag{7}$$

The left hand sides of equation (6) and (7) are considered as identical and implies that the RHS are also identical.

$$\frac{\bar{X}_{2U} - \bar{X}_U}{\bar{X}_1 - \bar{X}_L} = \frac{\bar{X}_L - \bar{X}_{2L}}{\bar{X}_1 - \bar{X}_L}$$

$$\Rightarrow \bar{X}_{2U} - \bar{X}_U = \bar{X}_L - \bar{X}_{2L}$$

$$\bar{X}_{2U} + \bar{X}_{2L} = \bar{X}_L + \bar{X}_U$$

$$2\bar{X}_1 = \bar{X}_U + \bar{X}_L \qquad (8)$$

$$(2)-(3) \Rightarrow$$

$$z_1^{\ L} - z_2^{\ U} = \frac{1}{\left(\sigma/\sqrt{n}\right)}\left[\bar{X}_1 - \bar{X}_L - (\bar{X}_{2U} - \bar{X}_U)\right]$$

$$\left(z_1^{\ L} - z_2^{\ U}\right)\sigma/\sqrt{n} = \bar{X}_1 - \bar{X}_L - \bar{X}_{2U} + \bar{X}_U$$

$$\left(X_1^{\ L} - z_2^{\ U}\right)\sigma/\sqrt{n} + \bar{X}_{2U} - \bar{X}_1 = \bar{X}_U - \bar{X}_L \qquad (9)$$

By solving equations (8) and (9) and get the value of $\bar{X}_U$

$$(8) \Rightarrow \bar{X}_U + \bar{X}_L = 2\bar{X}_1$$

$$(9) \Rightarrow \bar{X}_U - \bar{X}_L = (z_1^{\ L} - z_2^{\ U})\frac{\sigma}{\sqrt{n}} + \bar{X}_{2U} - \bar{X}$$

Adding these two equations and get,

$$2\bar{X}_U = [z_1^{\ L} - z_2^{\ U})\frac{\sigma}{\sqrt{n}} + \bar{X}_{2U} + \bar{X}_1$$

$$\bar{X}_U = \frac{1}{2}\left[\left(z_1^{\ L} - z_2^{\ U}\right)\frac{\sigma}{\sqrt{n}} + \bar{X}_{2U} + \bar{X}_1\right]$$

Substitute the value of $\bar{X}_U$ in equation (8) and get

$$\bar{X}_U + \bar{X}_L = 2\bar{X}_1$$

$$\Rightarrow \bar{X}_L = 2\bar{X}_1 - \bar{X}_U$$

$$= 2\bar{X}_1 - \frac{1}{2}\left[\left(z_1^{\ L} - z_2^{\ U}\right)\frac{\sigma}{\sqrt{n}} + \bar{X}_{2U} + \bar{X}_1\right]$$

$$= \frac{3}{2}\bar{X}_1 - \frac{1}{2}\left[\left(z_1^{\ L} - z_2^{\ U}\right)\frac{\sigma}{\sqrt{n}} + \bar{X}_{2U}\right]$$

$$\overline{X}_L = \frac{1}{2}\left[3\overline{X}_1 - \left\{\left(z_1{}^L - z_2{}^U\right)\frac{\sigma}{\sqrt{n}} + \overline{X}_{2U}\right\}\right]$$

The equations (5), (10) and (11) give the required values of n, $\overline{X}_U$ *and* $\overline{X}_L$ respectively.

Problem 2:

Suppose that a company would like to design a variable sampling plan for steel bars based on tensile strength with both an upper and lower acceptance limit. The company does not wish to accept steel bars with a tensile strength greater than 10100 psi or lower than 9900 psi more than 10% of the time. In other words $\overline{X}_{2L} = 9900, \overline{X}_{2U} = 10000 \; and \; \beta = 0.10$. Suppose that $\overline{X}_1 = 10000, \alpha = 0.05$ and the standard deviation is known to be 100 psi.

Solution:

By using the equations, the following equations are obtained

$$z_1{}^U = \frac{\overline{X}_U - \overline{X}_1}{\sigma/\sqrt{n}}$$

$$1.960 = \frac{\overline{X}_U - 10000}{100/\sqrt{n}}$$

$$z_1{}^L = \frac{\overline{X}_1 - \overline{X}_L}{\sigma/\sqrt{n}}$$

$$1.960 = \frac{10000 - \overline{X}_L}{100/\sqrt{n}}$$

$$z_2{}^L = \frac{\overline{X}_L - \overline{X}_{2L}}{\sigma/\sqrt{n}}$$

$$1.645 = \frac{\overline{X}_L - 9900}{100/\sqrt{n}}$$

$$z_2{}^U = \frac{\overline{X}_{2U} - \overline{X}_U}{\sigma/\sqrt{n}}$$

$$1.645 = \frac{10100 - \overline{X}_U}{100/\sqrt{n}}$$

Since the last equation can be derived from the other three equations, it will not be considered in the computations. Using the remaining three equations and solving for $\overline{X}_U, \overline{X}_L$ and n simultaneously, we obtain $\overline{X}_U = 10054, \overline{X}_L = 9946$ and n=13. In summary, the variables sampling plan is as follows. Obtain a random sample of 13 steel bars from a lot and compute the average tensile strength. If the sample average is higher than 10054 psi or lower than 9946 psi reject the lot, otherwise accept it.

(c) Variable Sampling Plans to Estimate the Process Average with a Single Specification Limits - σ unknown

Earlier, it was assumed that the variance is known. Consider the case where the variance is unknown. In this situation, a larger sample size will generally be required for a sampling plan than for a corresponding sampling plan in which the variance is known. The reason is that an additional amount of uncertainty is introduced into the sampling procedure, hence the need for a larger sample size. It is assumed that the items from a process or lot are still normally distributed.

Since the variance is unknown, the deviation of the sampling plan will be based on the t statistic. The t statistic was given equation

$$t_0 = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}$$

in slightly different form as

$$t = \frac{\overline{X} - \overline{X}_1}{S/\sqrt{n}}$$

$$S^2 = \frac{\sum_{i=1}^{n} X_i^{\,2} - n\overline{X}^2}{n-1}$$

and X_i is the measured quality characteristic of the i^{th} unit in the sample. Notice that the t statistic is expressed in terms of S, which in turn is expressed in terms of n, an unknown parameter. Therefore, the parameters of the sampling plan cannot be obtained in terms of sample standard S. Notice also that the sample standard deviation is based on the sample obtained from a lot. Therefore, the OC curve for a sampling plan will vary from lot to lot. Thus, for the reasons mentioned previously, a sampling plan cannot be derived unless an estimate of the standard deviation can be made.

Consider the design of sampling plan with characteristics $\overline{X}_1, \overline{X}_2, \alpha$ *and* β . Let $\hat{\sigma}$ be the estimate of the standard deviation of the process or lot. Duncan (1986) provides several OC curves for single limit sampling plans based on the t statistic for $\alpha=0.05$. To obtain the required sample size that meets the characteristics of our sampling plan, it is first necessary to compute λ as,

$$\lambda = \frac{\left|\overline{X}_1 - \overline{X}_2\right|}{\hat{\sigma}}$$

We can find n for $P_a = \beta$. Thus, the sampling plan is as follows:

Take a random sample of size n and compute the variate t to be

$$t = \frac{\overline{X} - \overline{X}_1}{S/\sqrt{n}}$$

If $t \le t_{0.05,n-1}$, reject the lot, otherwise accept it. Notice that even through an estimate of the standard deviation $\hat{\sigma}$ is used to derive a sampling plan, this estimate is not used during sampling. Instead, the standard deviation of a sample S is used to calculate for the variable t.

Problem 3:

Consider problem 1 with $\overline{X}_1 = 10000, \overline{X}_2 = 9950, \alpha = 0.05$ *and* $\beta = 0.10$. Assume that the standard deviation is unknown, but the company decides that a good estimate would be 90 psi (i.e. $\hat{\sigma} = 90$).

Solution:

By using the equation λ is computed to be

$$\lambda = \frac{|\overline{X}_1 - \overline{X}_2|}{\hat{\sigma}}$$

$$\lambda = \frac{|10000 - 9950|}{90} = 0.56$$

The t statistic for α=0.05 and 29 degrees of freedom is -1.699. Therefore, the sampling plan is as follows. Take a random sample of 30 steel bars from a lot and compute the sample average $\overline{X}$ and the sample standard deviation S. compute the variable t as

$$t = \frac{\overline{X} - 10000}{S/\sqrt{30}}$$

If $t \le -1.699$, reject the lot, otherwise accept it.

5.4. Variables Sampling Plans to Estimate the Lot Percent Nonconforming

It is assumed that the quality characteristic of interest is measurable on a continuous scale and that it is normally distributed, with a known and constant variance. By the normality assumption, a functional relationship exists between the lot percent nonconforming and the sample mean $\overline{X}$ and the standard deviation σ. This functional relationship is given by the standard normal deviates

$$Z_L = \frac{\overline{X} - L}{\sigma} \quad (1)$$

If a lower limit L is specified or

$$Z_U = \frac{U - \overline{X}}{\sigma} \quad (2)$$

If an upper limit U is specified. Therefore, the estimate of the lot percent nonconforming is the area under the normal curve that exceeds the normal deviate $Z_L or\ Z_U$. Notice that the normal deviate varies directly with the sample mean such that as the sample mean moves further away from the specification limit, the percent nonconforming decreases and vice versa.

Form 1 and Form 2

There are two methods by which a variables sampling plan that estimates the lot percent nonconforming can operate. They are called Form 1 (or the k-method) and Form 2 (or the M-method). Under Form 1, the standard normal deviate is computed by using equation (1) or (2) depending on whether a lower or an upper limit is specified. This method is called the k-method since the standard normal deviate $Z_L or\ Z_U$ is compared to critical value k. If $Z_L or\ Z_U$ is greater than or equal to the value of k, the lot is accepted. The operation of the variables sampling plan using this method is as follows. Take a random sample of size n from a lot. Compute the sample average $\overline{X}$ and the standard normal deviate $Z_L or\ Z_U$. If $Z_L or\ Z_U$ is greater than or equal to k, accept the lot, otherwise reject it.

The M-method is similar to k-method, but the estimation of the lot percent nonconforming is carried one step further. Let $\overline{X}$ be the sample mean the variance σ^2 is assumed to be known and constant. The normal deviate is computed as

$$Q_L = \frac{\overline{X} - L}{\sigma}\sqrt{\frac{n}{n-1}} \qquad (1)$$

If a lower limit is specified or

$$Q_U = \frac{U - \overline{X}}{\sigma}\sqrt{\frac{n}{n-1}} \qquad (2)$$

If an upper limit is specified. Notice that

$$Q_L = Z_L\sqrt{\frac{n}{n-1}} \qquad (3)$$

and

$$Q_U = Z_U\sqrt{\frac{n}{n-1}} \qquad (4)$$

The normal deviates $Q_L\ and\ Q_U$ provides a slightly better estimate of the lot percent nonconforming, since they are unbiased and have minimum variance. After $Q_L\ or\ Q_U$ is computed, the lot percent nonconforming p is estimated by determining the area under the normal curve that exceeds the quality index $Q_L\ or\ Q_U$. Call this estimate $\hat{p}$. If $\hat{p}$ exceeds a maximum allowable percent nonconforming M the lot is rejected, otherwise accepted. This method is called the M-method since the estimated lot percent nonconforming $\hat{p}$ is compared is as follows. Take a random sample of size n from a lot. Compute $\overline{X}$ and quality index

$Q_L \, or \, Q_U$. From the quality index, obtain the estimated lot percent nonconforming $\hat{p}$. If $\hat{p}$ is greater than M reject the lot, otherwise accept it.

(a) Variable Sampling Plans with a single specification limit for large sample size when σ unknown

In statistical hypothesis, when the population parameter is unknown, we have to use sample observations. Consider a sample of size n (>30) with the observations $x_1, x_2, ..., x_n$. By using the given observations, we have to calculate sample mean and sample standard deviation. Based on the computed sample statistics and specification limits, we have to frame variable sampling plan and estimate sample size n and critical value k. Let $\alpha, \beta, p_1 \, and \, p_2$ be the four characteristics which are used to frame the variable sampling plan. Let $z_\alpha, z_\beta, z_1 \, and \, z_2$ be the standard normal variables corresponding to the characteristics $\alpha, \beta, p_1 \, and \, p_2$. In 1947, Eisenhart, Hastay and Wallis have derived the sample size and critical value as

$$n = \left(1 + \frac{k^2}{2}\right)\left(\frac{z_\alpha + z_\beta}{z_1 - z_2}\right)^2 \tag{1}$$

and

$$k = \frac{z_\alpha z_2 + z_\beta z_1}{z_\alpha + z_\beta} \tag{2}$$

For framing standard variables we have to compute sample mean $\overline{X}$ and sample standard deviation S. In addition that we have to fix lower limit (L) and upper limit (U). If an upper limit is specified

$$z_U = \frac{U - \overline{X}}{S} \tag{3}$$

If a lower limit is specified

$$Z_L = \frac{\overline{X} - L}{S} \tag{4}$$

If either $z_U \, or \, z_L$ is greater than or equal to critical value k, then accept the lot. If it is not so, we reject the lot.

Construction of an OC Curve

As in the σ known case, the OC curve is constructed by plotting a range of lot percent nonconforming values p versus the probability of acceptance of each. Suppose that a single lower specification limit L is given. Let p be the percent nonconforming and let Z_p be the standard normal deviate in which the area is (1-p). The probability of acceptance for a p value is defined by calculating the normal deviate Z_a using the following equation:

$$Z_a = \frac{k - Z_p}{\sqrt{\frac{1}{n} + \frac{k^2}{2n}}} \tag{1}$$

The area exceeding Z_a can be determined. This area is the probability of acceptance value for p.

Problem 4:

Suppose a company is interested in setting up a variables sampling plan for thin-wall cylinders based on the tensile strength with $p_1 = 0.01, p_2 = 0.10, \alpha = 0.05 \, and \, \beta = 0.10$. Suppose also that the variance is unknown.

Solution:

The values of the standard normal deviates for $p_1, p_2, \alpha \; and \; \beta$ are $Z_1 = 2.327$, $Z_2 = 1.282, Z_\alpha = 1.645 \; and \; Z_\beta = 1.282$ respectively. Using the given equation to solve for k, we have

$$k = \frac{Z_\alpha Z_2 + Z_\beta Z_1}{Z_\alpha + Z_\beta}$$

$$k = \frac{(1.645)(1.282) + (1.282)(2.327)}{1.645 + 1.282} = 1.74$$

Using the given equation to solve for n, we have

$$n = \left(1 + \frac{k^2}{2}\right)\left(\frac{Z_\alpha + Z_\beta}{Z_1 - Z_2}\right)^2$$

$$n = \left(1 + \frac{(1.74)^2}{2}\right)\left(\frac{1.645 + 1.282}{2.327 - 1.282}\right)^2 = 19.72$$

Since n is integer valued, we round 19.72 to the next higher integer, which is 20. Therefore, the variables sampling plan is as follows. Take a random sample of 20 thin-wall cylinders and compute the average tensile strength and the sample standard deviation S. Using the given equation, to compute Z_L.

$$Z_L = \frac{\overline{X} - L}{S}$$

If $Z_L \geq 1.74$, accept the lot, otherwise reject it.

(b) Variable Sampling Plans with a single specification limit for large sample size when σ known

To derive a sampling plan to control the lot percent nonconforming, it is necessary to known four characteristics of the plan. They are α (the probability of rejecting a lot that does not meet the specified quality level), β (the probability of accepting a lot that does not meet the specified quality level), p_1 (the percent nonconforming value for which the probability of acceptance is high and p_2 (the percent nonconforming value for which the probability of acceptance is low). The OC curve of the sampling plan should pass through the two points $(1-\alpha, p_1)$ *and* (β, p_2). Let Z_α be the normal deviate for which the area under the standard normal curve is (1-α) and let Z_β be the normal deviate for which the area is (1-β). Let $\bar{X}$ be the sample average and suppose that a single lower limit L is specified and the standard deviation σ is known. Under Form 1, a lot will be accepted if the following holds

$$\frac{\bar{X}-L}{\sigma} \geq k$$

Adding and subtracting the term $\frac{\mu}{\sigma}$, we get

$$\frac{\bar{X}-L}{\sigma} + \frac{\mu}{\sigma} - \frac{\mu}{\sigma} \geq k$$

Such that

$$\frac{\bar{X}-\mu}{\sigma} + \frac{\mu - L}{\sigma} \geq k$$

Which can be written

$$\frac{\bar{X}-\mu}{\sigma} \geq k - \frac{\mu - L}{\sigma}$$

Multiplying both sides by $\sqrt{n}$, we get

$$\left(\frac{\bar{X}-\mu}{\sigma}\right)\sqrt{n} \geq \left(k - \frac{\mu - L}{\sigma}\right)\sqrt{n}$$

Which can be written

$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \geq \left(k - \frac{\mu - L}{\sigma}\right)\sqrt{n}$$

Let Z_1 *and* Z_2 be the standard normal deviates such that the areas exceeding these values are p_1 *and* p_2 respectively. Therefore,

$$\frac{\mu_1 - L}{\sigma} = Z_1$$

and

$$\frac{\mu_2 - L}{\sigma} = Z_2$$

So that

$$P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \geq (k - Z_1)\sqrt{n}\right) = \alpha$$

and

$$P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \geq (k - Z_2)\sqrt{n}\right) = \beta$$

Since $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ is normally distributed, with mean 0 and variance σ^2, we have

$$(k - Z_1)\sqrt{n} = Z_\alpha \tag{1}$$

and

$$(k - Z_2)\sqrt{n} = Z_\beta \tag{2}$$

To specify a sampling plan, it is necessary to know the sample size n and the critical value k. These two parameters can be obtained by solving equations (1) and (2) simultaneously. The resulting equations are,

$$n = \left(\frac{Z_\beta + Z_\alpha}{Z_1 - Z_2}\right)^2 \tag{3}$$

and

$$k = Z_1 - \frac{Z_\alpha}{\sqrt{n}} \tag{4}$$

If α is to be used as specified or

$$k = Z_2 + \frac{Z_\beta}{\sqrt{n}} \tag{5}$$

If β is to be used as specified.

Under Form 2, it is necessary to compute $k\sqrt{n/(n-1)}$, where k is the critical value obtained previously. The critical value M is determined as the area under the normal curve that is in excess of the value $k\sqrt{n/(n-1)}$.

Construction of an OC Curve

The OC curve for a sampling plan that estimates the lot percent nonconforming can be constructed by plotting a range of lot percent nonconforming values p versus the probability of acceptance of each value.

Suppose that a lower limit is specified. Let p be the lot percent nonconforming and let Z_p be the corresponding standard normal deviate in which the area is (1-p). Let $\overline{X}_p$ be the sample average such that the following holds

$$\frac{\overline{X}_p - L}{\sigma} = Z_p$$

Thus the probability of acceptance is given by the following expression

$$P\left(\frac{\overline{X}_p - \mu}{\sigma/\sqrt{n}} \geq (k - Z_p)\sqrt{n}\right) = a$$

So that

$$Z_a = (k - Z_p)\sqrt{n}$$

The area in excess of Z_a is the probability of acceptance for p.

Problem 5:

Suppose that it is desired to set up a sampling plan for thin-wall cylinders based on the tensile strength. The lower specification limit is 170 psi. The sampling plan should have the characteristic such that a lot will be accepted with a probability of 0.95 if it contains 1% defective and will be rejected with probability of 0.90 if it contains 10% defective. In other words $p_1 = 0.01, p_2 = 0.10, \alpha = 0.05$ *and* $\beta = 0.10$. Suppose that the standard deviation is known to be 10 psi.

Solution:

The values of the standard normal deviates for p_1, p_2, α *and* β are $Z_1 = 2.327$, $Z_2 = 1.282, Z_\alpha = 1.645$ *and* $Z_\beta = 1.282$ respectively. Using the given equation to solve for n, we obtain

$$n = \left(\frac{Z_\beta + Z_\alpha}{Z_1 - Z_2}\right)^2$$

$$n = \left(\frac{1.282 + 1.645}{2.327 - 1.282}\right)^2 = 7.845$$

Since n is integer valued, we round 7.845 to the next higher integer, which is 8. Using the given equation, keeping α at the 0.05 level and solving for k, we obtain

$$k = Z_1 - \frac{Z_\alpha}{\sqrt{n}}$$

$$k = 2.327 - \frac{1.645}{\sqrt{7.845}} = 1.74$$

Therefore, the variables sampling plan under Form 1 will be as follows. Take a random sample of eight thin-wall cylinders from the lot and compute the sample average tensile strength. Using the given equation, compute Z_L. If $Z_L \geq 1.74$, accept the lot, otherwise reject it.

Under Form 2, an extra computational step is needed to obtain an estimate of the percent nonconforming. The value of M is obtained by the first computing

$$k\sqrt{\frac{n}{(n-1)}}$$

$$k = 1.74\sqrt{\frac{8}{7}} = 1.86$$

The area under the normal curve of this value is the value of M, which is 0.0314. Therefore, the sampling plan under Form 2 is as follows. Take a random sample of eight cylinders from the lot. Using the given equation, compute Q_L.

$$Q_L = \frac{\overline{X} - L}{\sigma}\sqrt{\frac{n}{n-1}}$$

Determine $\hat{p}_L$, the area in excess of Q_L. If $\hat{p}_L \leq 0.0314$, accept the lot, otherwise reject it.

Exercises

1. What are sampling plans by variables? Give the assumptions under which such plans are defined.
2. Derive the sampling plan to estimate process parameter with double specification limits when σ is known.
3. Derive the sampling plan to estimate process average with a single specification limit when σ is known.
4. What is meant by acceptance sampling by variables? Also, state its importance in industries.
5. Derive the variable sampling plans to estimate the process average with a single specification limits when σ is unknown.

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UNIT 6 RECTIFYING SAMPLING PLANS

Structure

6.1 INTRODUCTION

In Unit 5, you have learnt about the acceptance sampling plans and how these are implemented in industry. You have also learnt about different terms related to it such as acceptance quality level (AQL), lot tolerance percent defective (LTPD), producer's risk and consumer's risk. In an acceptance sampling plan, if the consumer rejects the lot on the basis of the information provided by the inspected sample and the producer is not able to sell the rejected lot, he/she suffers loss. If this continues, the producer might face a huge loss and not be able to continue with production. He/she may even have to shut it down. To avoid such situations, the concept of the rectifying sampling inspection plan was introduced.

In this unit, you will learn about the rectifying sampling plans for attributes. In Sec. 6.2, we discuss what is the rectifying sampling plan and how it implement in industry. In Secs. 6.3 to 6.6, we introduce some more parameters such as average outgoing quality (AOQ), operating characteristic (OC) curve, average sample number (ASN) and average total inspection (ATI). In the next unit, you will study single sampling plans.

Objectives

After studying this unit, you should be able to:

- explain why rectifying sampling plans are needed;
- describe a rectifying sampling plan;
- define average outgoing quality (AOQ) and the average outgoing quality limit (AOQL);
- describe the operating characteristic (OC) curve; and
- define average sample number (ASN) and average total inspection (ATI).

6.2 RECTIFYING SAMPLING PLAN

In an acceptance sampling plan, the consumer accepts the lot if the acceptance criteria are satisfied. Otherwise, he/she rejects the lot. This increases the producer's risk. Rectifying sampling plans are used to reduce the producer's risk. In such a plan, the entire lot is not rejected. Instead, each and every unit/item of the lot is inspected. It means that 100% inspection of the rejected lot is carried out and the defective units found in the lot are replaced by non-defective units. This procedure is known as **rectifying** or **screening** or **detailing** the rejected lots.

The sampling plan in which 100% inspection is carried out for rejected lots is called the **rectifying sampling inspection plan**. It is also called the **rectifying sampling plan** or **rectifying inspection plan**. Let us explain the procedure for implementing this plan.

6.2.1 Implementation of Rectifying Sampling Plan for Attributes

Suppose that lots of the same size (N) are received from the supplier or the final assembly line and submitted for inspection one at a time. The procedure for implementing the rectifying sampling plan to arrive at a decision about the lot is described in the following steps:

Step 1: We draw a random sample of size n from the lot received from the supplier or the final assembly.

Step 2: We inspect each and every unit of the sample and classify it as defective or non-defective on the basis of certain criteria. At the end of the inspection, we count the number of defective units found in the sample.

Step 3: We compare the number of defective units found in the sample with the acceptance criteria.

Step 4: If acceptance criteria are satisfied, we accept the entire lot by replacing all defective units **in the sample** by non-defective units. If the criteria are not satisfied, we accept the lot by inspecting **the entire lot** and replacing all defective units in it by non-defective units.

The steps described above are shown in Fig. 6.1.

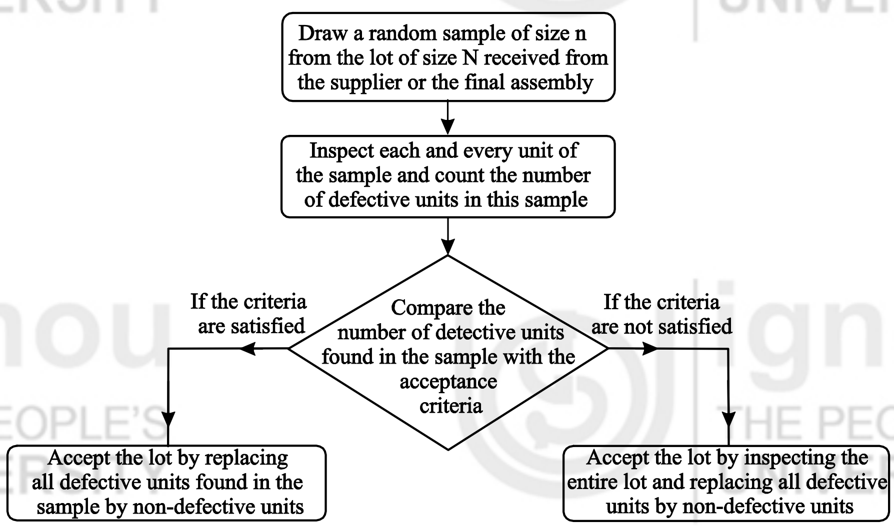

Fig. 6.1: Procedure for implementation of the rectifying sampling plan.

Let us explain these steps further with the help of an example.

Example 1: Suppose a cricket ball manufacturing company supplies lots of 200 balls. To check the quality of the lots, the buyer and the manufacturing company decide that a sample of size 20 will be drawn from each lot. The lot would be accepted if the inspected sample contains at most one defective ball. Otherwise, the lot would be rejected. If they use the rectifying sampling plan, explain the procedure for implementing it.

Solution: For implementing the rectifying sampling plan, the buyer draws 20 balls from each lot, inspects each and every ball of the sample and classifies it as defective or non-defective on the basis of certain criteria. At the end of the inspection, he/she counts the number of defective balls found in the sample and compares the number of defective balls with the acceptance criteria. If the number of defective balls in the sample is less than or equal to the acceptance number, he/she accepts the lot by replacing all defective balls in the sample by non-defective balls. If the number of defective balls is greater than the acceptance number, he/she carries out 100% inspection for this lot instead of rejecting the lot. The defective balls found in the lot are replaced by non-defective balls and the lot is accepted.

You may like to explain the procedure for implementing a rectifying sampling plan yourself. Try the following exercise.

E1) A hospital receives disposable injection syringes in lots of 2000. A single sampling plan with n = 25 and c = 2 is being used for accepting the lots. Explain the procedure for implementing the rectifying sampling plan in this case.

So far you have learnt about the rectifying sampling plan. In Secs. 6.3 to 6.6, we explain various terms associated with the rectifying sampling plans.

6.3 AVERAGE OUTGOING QUALITY (AOQ)

In a sampling inspection plan, the items or units produced/manufactured by the producer are formed in lots. The average quality level of the lots is set by the producer and the consumer through negotiation and the producer sends the lots to the consumer for inspection. The quality of the lots before the inspection is known as **incoming quality** and the quality of the lots which have been accepted after the inspection is known as **outgoing quality**. In an acceptance sampling plan, the lots are either accepted or rejected. So the outgoing quality is the same as the incoming quality. However, in a rectifying sampling inspection plan, the rejected lots are rectified or screened. So the outgoing quality will differ from the incoming quality. Therefore, the concept of **average outgoing quality** (AOQ) is particularly useful for the evaluation of a rectifying sampling plan.

You have learnt in Sec.6.2 that the consumer takes a sample from each lot to check its quality. He/she inspects each item or unit of the sample for defects. If the number of defective units in the sample is less than the acceptance number, the lot is accepted by replacing all defective units found in the sample by non-defective units. If the number of defective units is greater than the acceptance number, each and every unit of the lot is inspected. It means that 100% inspection is carried out for each rejected lot and all defective units found in the lot are replaced by non-defective units. Therefore, these lots are accepted after 100% inspection with zero percent defective. As a result, the accepted stores will consist of lots of varying quality level, ranging from quality levels lower than acceptance quality level to lots with zero defective. Therefore, we need to define the concept **average outgoing quality**.

When all lots are considered together, their average quality level may be considerably different from the incoming quality. The average outgoing quality (AQL) is defined as follows:

The expected quality of the lots after the application of sampling inspection is called the **average outgoing quality**. It is calculated as follows:

$$AOQ = \frac{\text{Number of defective units in the lot after inspection}}{\text{Total number of units in the lot}} \quad \ldots(1)$$

But we do not know the number of defective units in the entire lot after the inspection of samples. So we have an alternative way of calculating AOQ. If N is the size of each lot, n is the sample size inspected in each lot, p is the incoming quality of the lots and P_a is the probability of accepting the lot of incoming quality p, AOQ for single sampling plan is given by

$$AOQ = \frac{p(N-n)P_a}{N} \quad \ldots(2)$$

We shall derive this formula in Unit 7. The concept of AOQ is shown in Fig. 6.2.

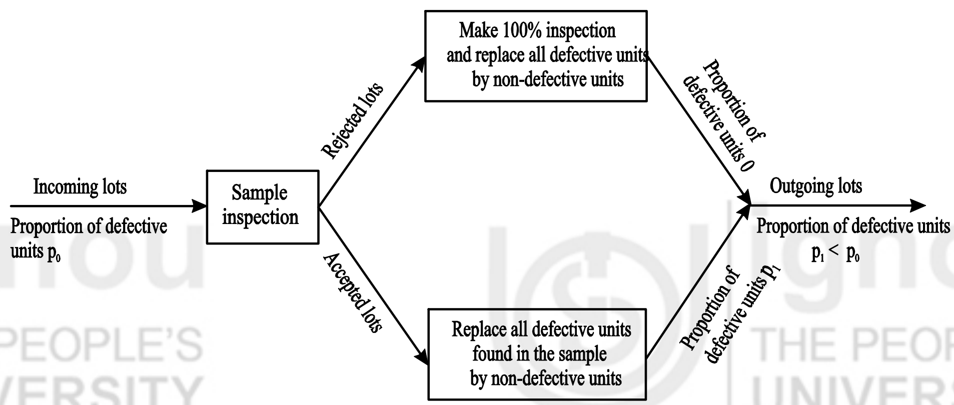

Fig. 6.2: Concept of average outgoing quality.

For the acceptance sampling plan in which rectification is not done, the AOQ is the same as the incoming quality.

If we draw a graph of AOQ versus the incoming quality, the curve so obtained is known as the **AOQ curve**. A typical AOQ curve is shown in Fig. 6.3.

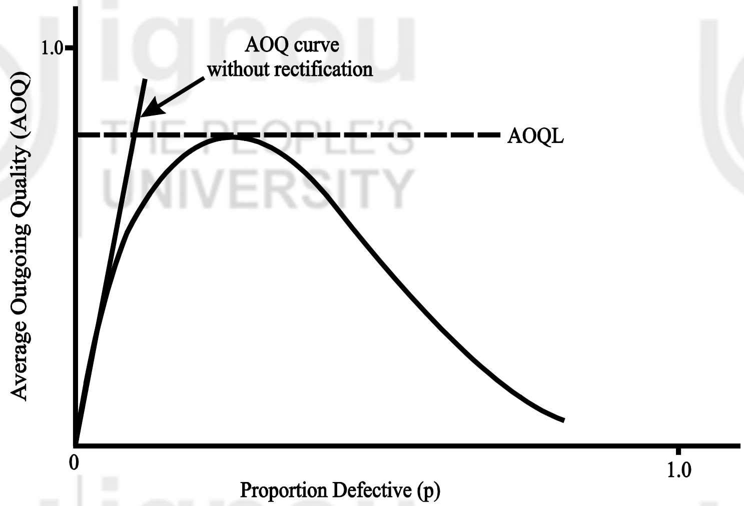

Fig. 6.3: The AOQ curve.

If we analyse the AOQ curve (shown in Fig. 6.3), we observe that for p = 0, the incoming lots have no defective units and hence there are no defective units in the outgoing lots. The consumer accepts all lots. So in this case, AOQ = 0. When the incoming quality is good, a large proportion of the lots will be accepted by the rectifying sampling plan and only a smaller fraction will be screened. Hence, the outgoing quality will be good. However, when the incoming quality is not good, a large proportion of the lots will be screened. In such cases, the outgoing quality will also be good because defective units or items will either be replaced or rectified. Between these extremes, the AOQ increases up to a maximum and then decreases. The maximum value of AOQ represents the worst possible average for the outgoing quality. It is known as the **average outgoing quality limit (AOQL)**. The AOQL tells us that no matter how poor the incoming quality is, on an average, the outgoing quality will never be poorer than AOQL.

Let us take up an example based on AOQ.

Example 2: Suppose a cricket ball manufacturing company formed lots of 500 balls. To check the quality of the lots, the buyer draws 20 balls from each lot and accepts the lot if the sample contains at most one defective ball. Otherwise, he/she rejects it. Suppose the quality of the submitted lots is 0.03. Calculate the AOQ for this sampling plan. If rejected lots are screened and all defective balls are replaced by non-defective ones, calculate the AOQ again.

Solution: It is given that

$$N = 500, n = 20, c = 1 \text{ and } p = 0.03$$

We know that in an acceptance sampling plan, the lots are either accepted or rejected and the average outgoing quality is the same as the incoming quality. Therefore, in the first case, when the lot is either accepted or rejected, the AOQ is the same as the quality of the submitted lot, i.e., AOQ = 0.03.

In the second case, when the rejected lots are screened and all defective balls are replaced by non-defective ones, we calculate the AOQ using equation (2).

For calculating the AOQ in this case, we have to calculate the probability of accepting a lot (P_a). Recall the method of fining it described in Sec. 5.5 of Unit 5.

If X represents the number of defective balls in the sample, the buyer accepts the lot if $X \le c$. Here c = 1. Therefore, the probability of accepting the lot is given by

$$P_a = P\left[X \le c = 1\right] = \sum_{x=0}^{1} P\left[X = x\right]$$

Since $N \ge 10n$, we can use the binomial distribution and Table I given at the end of this block to obtain P_a.

From Table I, for n = 20, x = c = 1 and p = 0.03, we have

$$P_a = P\left[X \le 1\right] = \sum_{x=0}^{1} {}^nC_x\, p^x \left(1-p\right)^{n-x} = 0.8802$$

On putting the values of N, n, p and P_a in equation (2), we get

$$AOQ = \frac{p\left(N-n\right)P_a}{N} = \frac{0.03 \times \left(500-20\right) \times 0.8802}{500} = 0.025$$

You may like to calculate the AOQ in the following exercises.

E2) Suppose, in Example 2, the quality of the submitted lots is 0.01, calculate AOQ for this sampling plan.

E3) Suppose, in E1, the quality of the submitted lot is 0.05, calculate AOQ for this sampling plan.

We now describe another feature of the sampling plan known as the operating characteristic curve.

6.4 OPERATING CHARACTERISTIC (OC) CURVE

You have learnt that in acceptance sampling, the lot is either rejected or accepted on the basis of conclusions drawn from the sample. Sometimes, a good lot may be rejected and a bad lot may be accepted because we infer the quality of all units in the lot on the basis of a sample (small part of the lot).

So we require an acceptance sampling plan which ensures that good lots are always accepted and the bad lots are always rejected. It means that the plan should perfectly discriminate between good and bad lots. Therefore, we introduce a tool called the **operating characteristic curve**, which measures the discriminating ability of a plan. The operating characteristic (OC) curve of an acceptance sampling plan reflects the ability of the plan to distinguish between good and bad lots.

The OC curve for a sampling plan is a graph of the probability of accepting the lot versus the proportion defective (proportion of defective units) in the lot. It is plotted by taking the proportion defective (p) along the X-axis and the probability of accepting the lot along the Y-axis.

When the acceptance sampling plan is used, there is a conflict of interest between the producer and the consumer. The producer wants that all acceptable lots to be accepted and the consumer wants that all unsatisfactory lots to be rejected. A sampling plan which perfectly discriminates between good and bad lots is called an **ideal sampling plan**. The OC curve for an ideal sampling plan is called an **ideal OC curve** and is shown in Fig. 6.4.

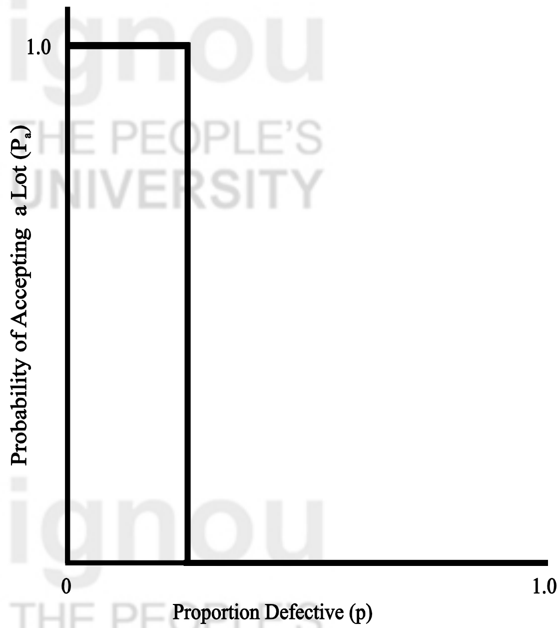

Fig. 6.4: Ideal OC curve.

The ideal OC curve runs horizontally at a probability of acceptance $P_a = 1.0$ until a level of lot quality that is considered **bad** is reached. At that point, the curve drops vertically to a probability of acceptance $P_a = 0$. Then the curve runs horizontally again for all lot proportion defectives greater than the undesirable level. If such a sampling plan could be employed, all lots of **bad** quality would be rejected and all lots of **good** quality would be accepted. But, the ideal OC curve can almost never be obtained in practice. In theory, it could be realised by 100% inspection, if the inspections were error free. But for 100% inspection, the cost is high and the time of inspection is much more. So we use sampling plans. The ideal OC curve shape can be approached by increasing sample size. A typical OC curve is shown in Fig. 6.5.

The OC curve shows the acceptance probability of a lot of certain quality. It represents the relationship between the probability of acceptance and the quality of the lot. If the lot has no defective units, it is certain to be accepted and if it has all defective units then it is equally certain to be rejected. Hence, the OC curve has to start with $P_a = 1$ when $p = 0$. As the proportion defective (p) of the lot increases, the probability P_a of lot acceptance decreases until it reaches 0. Thus, $P_a = 0$ when $p = 1$.

The OC curve for a particular combination of n and c shows how well the plan discriminates between good and bad lots. In Fig. 6.5, we have shown an OC curve for n = 100 and c = 2. In this plan, if 0, 1, or 2 defective units are found in the sample of size n = 100, the lot would be considered acceptable and if more than 2 defective units are found, it would be rejected. Suppose, the actual lot quality is 0.01, i.e., there are 1 percent defective units in the lot, the OC curve shows that the probability of accepting the lot is approximately 0.99. It means that this sampling plan accepts the lot about 99% of the time and rejects the lot about 1% of the time. However, if the actual quality of a lot is 0.05, i.e., there are 5 percent defective units, the OC curve shows that the probability of accepting the lot is approximately 0.19.

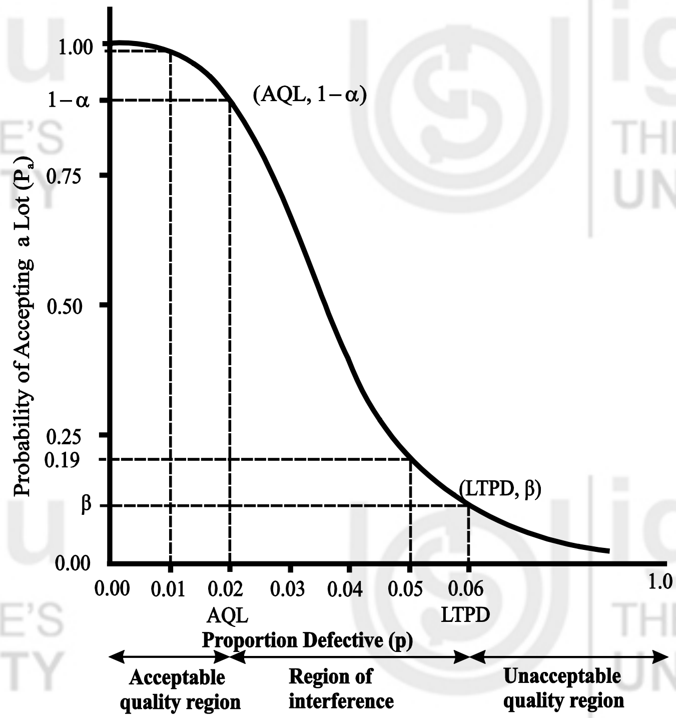

Fig. 6.5: the OC curve.

Hence, if the actual quality of a lot is good, the plan provides high probability of acceptance and if its quality is poor, the probability of acceptance of the lot is low. Thus, the OC curve shows how well a given plan discriminates between good and bad quality.

The OC curve of a sampling plan passes through two points agreed upon by the producer and the consumer. These points are (AQL, 1– α) and (LTPD, β) where the average quality level (AQL) and the lot tolerance percent defective (LTPD) are decided by the producer and the consumer after negotiation with each other. You know from Unit 5 that α and β are the producer's risk and the consumer's risk, respectively. Since the OC curve represents the probability of accepting a lot, instead of taking α (probability of rejecting a lot of quality AQL) we take 1 – α (probability of accepting a lot of quality AQL).

In practice, the performance of acceptance sampling plan for discriminating good and bad lots depends mainly on the sample size (n) taken from the lot and the acceptance number (c) that can be permitted in the sample. Since n and c are different in different sampling plans, the OC curve will be different for different sampling plans. Let us see how the sample size (n) and the acceptance sampling number (c) affect the OC curve.

Effect of increasing the sample size n on OC curve

Study Fig. 6.6. It shows the effect of increasing sample size on OC curve. Here we have three OC curves for three sampling plans with n = 100, c = 1; n = 200, c = 2 and n = 300, c = 3. From Fig. 6.6, we note that even though each plan uses the same percent defective which can be allowed for an acceptance lot (1/100, 2/200, 3/300 = 0.01, i.e.,1 percent), the OC curve becomes steeper and lies closer to the origin as the sample size increases.

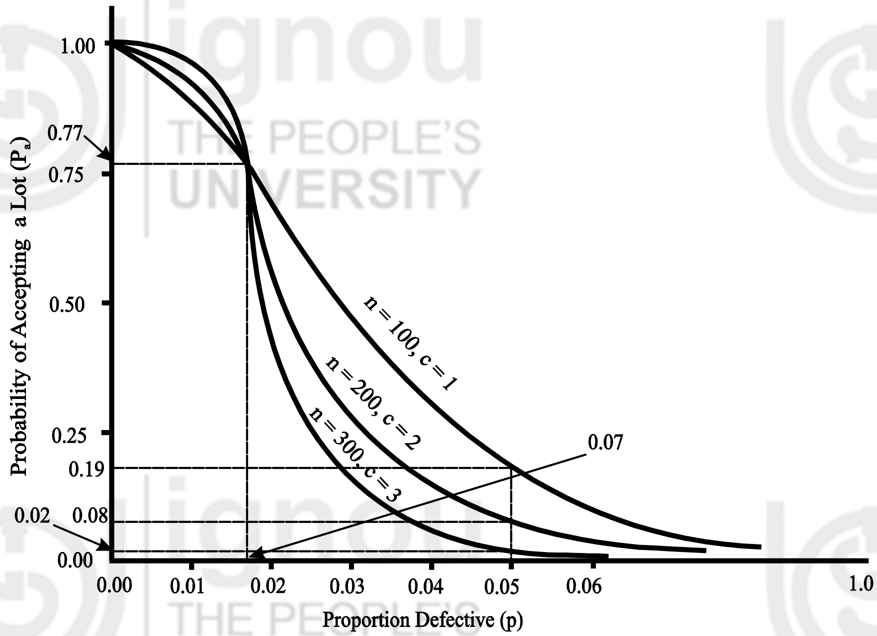

Fig. 6.6: The OC curves for n = 100, c = 1; n = 200, c = 2 and n = 300, c = 3.

The greater the slope of the OC curve, greater the discriminatory power.

If we compare the discrimination power of the three plans, we see that all three plans would accept lots with about 0.017, i.e., 1.7 percent defective units, about 77 percent of the time. However, if the actual quality falls to 5 percent defective units, the plan n = 100, c = 1 accepts the lot about 19 percent of the time, n = 200, c = 2 about 8 percent of the time and n = 300, c = 3 about 2 percent of the times. Therefore, the plans with larger sample sizes have definitely more discrimination power. It means that the plans with larger sample sizes are more effective in protecting the consumer against the acceptance of relatively bad lots and also give protection to the producer against rejection of relatively good lots.

Effect of decreasing the acceptance number (c) on OC curve

In Fig. 6.6, you have seen that the OC curve changes as the sample size changes. Generally, changing the acceptance number does not dramatically change the slope of the OC curve. As the acceptance number is decreased, the OC curve shifts to the left. For the same n, plans with smaller values of c provide discrimination at lower levels of the lot proportion defective than the plans with larger values of c. You can see this in Fig. 6.7, which shows the effect of acceptance number on the OC curve. It has three OC curves for n = 200 and c = 1, 2 and 3.

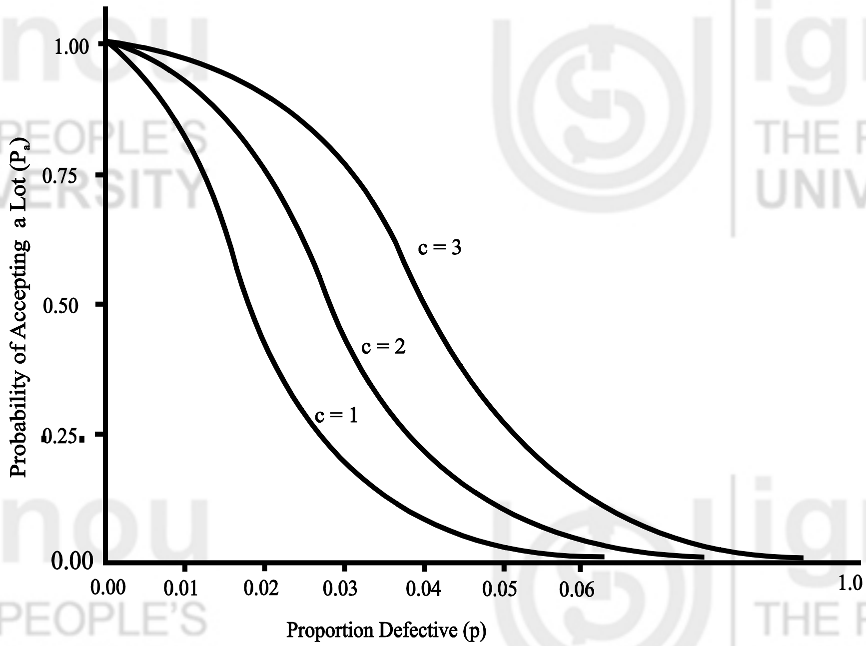

Fig. 6.7

Note that in Fig. 6.7, the effect of decreasing c is that the OC curve becomes steeper. Thus, when we decrease the acceptance number (c), the sampling plan becomes **tighter or stricter**. It means that the plan with smaller acceptance number has more discrimination power.

Now that you know what the OC curve of a sampling plan is and why it is used, let us summarize its properties.

Properties of the Operating Characteristic Curve

The OC curve possesses the following properties:

i) The OC curve of an acceptance sampling plan shows the ability of the plan to distinguish between good and bad lots.

ii) The OC curve with larger sample size is steeper. It means that the plan with larger sample size has more discrimination power. The plan with larger sample size protects the consumer against the acceptance of relatively bad lots and also gives protection to the producer against rejection of relatively good lots.

iii) The OC curve for a fixed sample size and smaller acceptance sample number is steeper. It means that the plan with a smaller acceptance number has more discrimination power.

iv) The OC curves with fixed sample sizes give constant quality protection, whereas the OC curves with relative sample sizes give different quality protections.

We now discuss another useful concept of sampling plans called the **average sample number (ASN)**.

6.5 AVERAGE SAMPLE NUMBER (ASN)

In acceptance sampling plan, the decision of acceptance or rejection of a lot is based on the information provided by the sample (s) drawn from the lot. So the average sample number can be defined as follows:

The **average sample number (ASN)** is defined as the average (expected) number of sample units per lot, which is required to arrive at a decision about the acceptance or rejection of the lot under the **acceptance sampling plan**.

The curve drawn between the ASN and the lot quality (p) is known as the **ASN curve**.

In a single sampling plan, we take the decision of acceptance or rejection of the lot on the basis of only a single sample of size n. Hence, the ASN in a single sampling plan is simply the sample size n, which means that ASN is constant. Therefore, the ASN curve for a single sampling plan is a straight line as shown in Fig. 6.8.

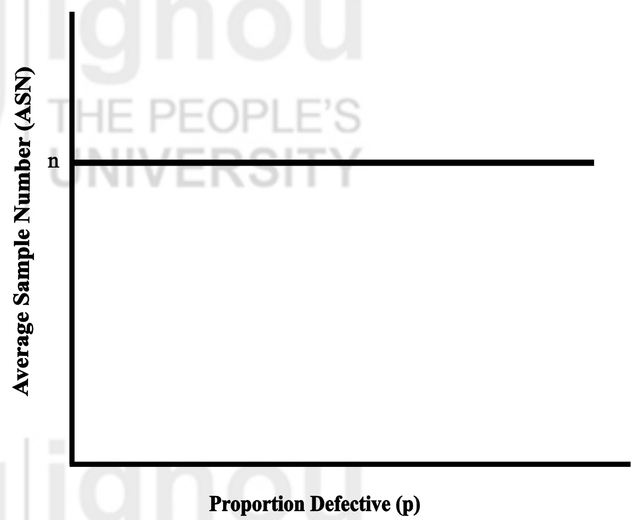

Fig. 6.8: The ASN curve for a single sampling plan.

ASN for a double sampling plan varies with the quality of the lot and is a function of the actual proportion defective (p) in the lot. We shall discuss the method of finding ASN and ASN curve for a double sampling plan in Unit 8.

The concept of average sample number is considered for the acceptance sampling plan. The average total inspection (ATI) plays the same role in a rectifying sampling plan.

6.6 AVERAGE TOTAL INSPECTION (ATI)

The average total inspection (ATI) for a rectifying sampling plan is defined as follows:

The average number of units inspected per lot under the rectifying sampling plan is called the average total inspection (ATI).

In other words,

The **average total inspection (ATI)** is the number of units inspected per lot to take the decision for acceptance or rejection of the lot under rectifying sampling plan calling for 100% inspection of the rejected lots.

If the lot quality is p, the average total inspection (ATI) for a single sampling plan is given by

$$ATI = n + (1 - P_a)(N - n) \qquad \ldots(3)$$

We shall derive this formula in Unit 7.

Therefore,

ATI = ASN + (average number of units inspected in the rejected lots)

Thus, if the lot is accepted on the basis of the rectifying sampling plan, then

ATI = ASN

Otherwise,

ATI > ASN

If a lot contains no defective unit, it will obviously be accepted by the sampling plan and only n items will be inspected. Therefore, in this case ATI will be equal to the sample size n. If all units of the lot are defective, the lot will be rejected and 100% inspection of the lot will be called. Therefore, in this case the ATI will be equal to the lot size N. If the lot quality lies between 0 and 1, i.e., $0 < p < 1$, the ATI will lie between the sample size n and the lot size N. This means that ATI is a function of the lot quality p.

The curve drawn between ATI and the lot quality (p) is known as **ATI curve**. A typical ATI curve for a single sample plan is shown in Fig. 6.9 for N = 2000, n = 30 and c = 2.

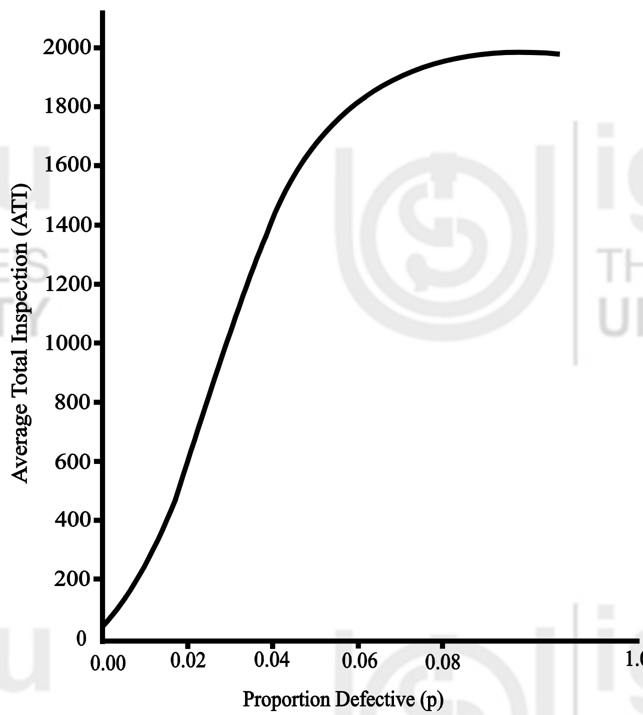

Fig. 6.9: The ATI curve.

We shall discuss the methods of obtaining ATI and ATI curves for single and double sampling plans in Units 7 and 8.

Let us now compute ASN and ATI for a real life situation.

Example 3: A hospital receives disposable injection syringes in lots of 2000. A single sampling plan with n = 25 and c = 2 is being used for inspection. Calculate the average sample number (ASN). Suppose the incoming lots contain 5% defective units. Calculate the average total inspection (ATI) for this plan, if the rejected lots are 100% inspected and all defective syringes are replaced by non-defective syringes.

Solution: It is given that

$$N = 2000, n = 25, c = 2, p = 0.05$$

In the first case, the decision of acceptance or rejection of the lot is taken only on a single sample of size n = 25. Therefore, the ASN for this plan is simply the sample size, i.e., ASN = n = 25.

We calculate ATI using equation (3).

We first calculate the probability of accepting a lot of quality p = 0.05.

If X represents the number of defective injection syringes in the sample, the hospital accepts the lot if $X \le c = 2$. Therefore, the probability of accepting the lot is given by

$$P_a = P[X \le 2] = \sum_{x=0}^{2} P[X = x]$$

Since $N \ge 10n$, we can obtain this probability by using Table I.

From Table I, for n = 25, x = c = 2 and p = 0.05, we have

$$P_a = P[X \le 2] = \sum_{x=0}^{2} {}^{n}C_x \, p^x (1-p)^{n-x} = 0.8729$$

On putting the values of N, n and P_a in equation (3), we get

$$ATI = n + (1 - P_a)(N - n)$$

$$= 25 + (1 - 0.8729)(2000 - 25)$$

$$= 25 + 251.02 = 276.02 \simeq 277$$

It is time for you to do the following exercises based on ASN and ATI.

E4) Explain the following terms:

i) Average sample number (ASN), and

ii) Average total inspection (ATI).

E5) A computer manufacturer purchases computer chips from a company in lots of 400. Twenty computer chips are sampled from each lot at random and inspected for defects. The computer manufacturer accepts the lot if inspected sample contains at most one defective chip. Otherwise, he rejects it. Suppose the incoming lots contain 3% defective chips. Calculate ASN for this plan. If rejected lots are screened and all defective computer chips are replaced by non-defective ones, calculate AOQ for this plan.

You have studied four curves, namely, AOQ, OC, ASN and ATI in Secs. 6.3 to 6.6. Their main purpose is to assess the efficiency and compare the costs of sampling plans. The AOQ and OC curves serve to assess protection of the consumer and the producer in terms of their risks given by the sampling plan and also give an idea of the efficiency of the sampling plan. The ASN and ATI curves give an indication of the cost of inspection. With the help of these curves, we arrive at a balance between the costs of inspection and the degree of protection. If inspection (sampling) costs are more than the costs of wrong decisions (without the aid of such inspection), then that particular inspection plan cannot be justified on economic considerations.

We now end this unit by giving a summary of what we have covered in it.

6.7 SUMMARY

1. An acceptance sampling plan in which rejected lots are 100% inspected is called a **rectifying sampling plan**.
2. The quality of the lots before inspection is known as **incoming quality** and the quality of the lots which have been accepted after the inspection is known as **outgoing quality**.
3. The expected quality of the lots after the application of sampling inspection is called **average outgoing quality (AOQ)** and is given by

$$AOQ = \frac{\text{Number of defective units in the lot after inspection}}{\text{Total number of units in the lot}}$$

4. The maximum value of AOQ represents the worst possible average for the outgoing quality and is known as the **average outgoing quality limit (AOQL)**.
5. The **OC curve** for a sampling plan is a graph of the probability of accepting the lot versus the proportion or fraction defective in the lot. It shows the ability of the plan to distinguish between good and bad lots.
6. A sampling plan which perfectly discriminates between good and bad lots is called an **ideal sampling plan** and the OC curve for this sampling plan is called an **ideal OC curve**.
7. The **precision** with which a sampling plan differentiates between good and bad lots increases with the **size of the sample**: The **greater the slope** of the OC curve, the **greater is the discrimination power** of the plan. The plan with a **smaller acceptance number** has **more discrimination power**.
8. The **average sample number (ASN)** is defined as the average (expected) number of sample units per lot, which is required to arrive at a decision about the acceptance or rejection of the lot under acceptance sampling plan. The curve drawn between ASN and lot quality (p) is known as the **ASN curve**.
9. The average number of units inspected per lot under the rectifying sampling plan is called the **average total inspection**. The curve drawn between ATI and lot quality (p) is known as the **ATI curve**.

6.8 SOLUTIONS/ANSWERS

E1) To check the quality of the lot, the quality inspector of the hospital draws 25 syringes randomly from each lot of disposable injection syringes and inspects each and every syringe of the sample. He/she classifies each syringe as defective or non-defective on the basis of certain defects. At the end of the inspection, he/she counts the number of defective syringes found in the sample and compares the number of defective syringes found in the sample with the acceptance number. If the number of defective syringes in the sample is greater than $c = 2$, say, 3, instead of rejecting the lot, he/she calls for 100% inspection of this lot and replaces all defective syringes found in the lot by non-defective syringes. Then he/she accepts the lot. If the number of defective syringes is less than or equal to c = 2, say, 1, he/she accepts the lot by replacing all defective syringes in the sample by non-defective syringes.

E2) It is given that

$$N = 500, n = 20, c = 1 \text{ and } p = 0.01$$

We calculate AOQ for the single sampling plan using equation (2). We first calculate P_a. Since $N \geq 10n$, we can obtain this probability by using Table I.

From Table I, for n = 20, x = c = 1 and p = 0.01, we have

$$P_a = P[X \leq 1] = \sum_{x=0}^{1} {}^nC_x \, p^x (1-p)^{n-x} = 0.9831$$

On putting the values of N, n, p and P_a in equation (2), we get

$$AOQ = \frac{p(N-n)P_a}{N} = \frac{0.01 \times (500-20) \times 0.9831}{500}$$

$$= \frac{4.7189}{500} = 0.009$$

E3) It is given that

$$N = 2000, n = 25, c = 2 \text{ and } p = 0.05$$

We calculate AOQ for the single sampling plan using equation (2).

Since here $N \geq 10n$, the probability of accepting the lot can be obtained as in E2.

$$\therefore \; P_a = P[X \leq 2] = \sum_{x=0}^{2} {}^nC_x \, p^x (1-p)^{n-x} = 0.8729$$

On putting the values of N, n, p and P_a in equation (2), we get

$$AOQ = \frac{p(N-n)P_a}{N} = \frac{0.05 \times (2000-25) \times 0.8729}{2000}$$

$$= \frac{86.1989}{2000} = 0.043$$

E4) Refer to Sec. 6.5 for (i) and Sec. 6.6 for (ii).

E5) In this sampling plan, the decision of acceptance or rejection of the lot is taken only using a single sample of size n = 20. Therefore, the ASN for this plan is simply the sample size n = 20.

It is given that

$$N = 400, n = 20, c = 1, p = 3\% = 0.03$$

We calculate ATI for the single sampling plan using equation (3).

Since $N \geq 10n$, we can obtain the probability of accepting a lot of quality p = 3% = 0.03 using Table I.

From Table I, for n = 20, x = c = 1 and p = 0.03, we have

$$P_a(p) = P[X \le 1] = \sum_{x=0}^{1} {}^{n}C_x\, p^x (1-p)^{n-x} = 0.8802$$

On putting the values of N, n and P_a in equation (3) we get

$$ATI = n + (1 - P_a)(N - n)$$

$$= 20 + (1 - 0.8802)(400 - 20)$$

$$= 20 + 45.524 = 65.524 \simeq 66$$

UNIT 7 SINGLE SAMPLING PLANS

Structure

7.1 INTRODUCTION

In Units 5 and 6, you have learnt the various features of a sampling inspection plan such as AQL, LTPD, producer's risk, consumer's risk, OC curve, ASN, ATI, etc. The same features characterise different types of sampling plans. In Unit 5, you have also learnt that the main types of acceptance sampling plans for attributes are:

i) Single sampling plan,

ii) Double sampling plan,

iii) Multiple sampling plan, and

iv) Sequential sampling plan

In this unit, we focus on the **single sampling plans for attributes**. In Sec. 7.2, we explain the single sampling plan and its implementation. We describe various features of the single sampling plan such as the operating characteristic (OC) curve, producer's risk, consumer's risk, average sample number (ASN) and average total inspection (ATI) in Secs. 7.3 to 7.6. Finally, we describe the design of the single sampling plans in Sec. 7.7. In the next unit, we shall discuss the double sampling plans for attributes.

Objectives

After studying this unit, you should be able to:

- describe a single sampling plan;
- compute the probability of accepting or rejecting a lot of given incoming quality in a single sampling plan;
- construct the operating characteristics (OC) curve of a single sampling plan;
- compute the consumer's risk and producer's risk in a single sampling plan;

- compute the average sample number (ASN) and the average total inspection (ATI) for a single sampling plan; and
- design single sampling plans.

7.2 SINGLE SAMPLING PLAN

A sampling plan in which a decision about the acceptance or rejection of a lot is based on a single sample that has been inspected is known as a **single sampling plan**. For example, suppose a buyer purchases cricket balls in lots of 500 from a company manufacturing cricket balls. To check the quality of the lots, the buyer draws a random sample of size 20 from each lot and takes a decision about accepting or rejecting of the lot on the basis of the information provided by this sample. Since the buyer takes the decision about the lot on the basis of a single sample, this sampling plan is a single sampling plan.

A single sampling plan requires the specification of two quantities which are known as **parameters** of the single sampling plan. These parameters are

n – size of the sample, and

c – acceptance number for the sample.

Let us suppose that the lots are of the same size (N) and are submitted for inspection one at a time. The procedure for implementing the single sampling plan to arrive at a decision about the lot is described in the following steps:

Step 1: We draw a random sample of size n from the lot received from the supplier or the final assembly.

Step 2: We inspect each and every unit of the sample and classify it as defective or non-defective. At the end of the inspection, we count the number of defective units found in the sample. Suppose the number of defective units found in the sample is d.

Step 3: We compare the number of defective units (d) found in the sample with the stated acceptance number (c).

Step 4: We take the decision of acceptance or rejection of the lot on the basis of the sample as follows:

Under acceptance sampling plan

If the number of defective units (d) in the sample is less than or equal to the stated acceptance number (c), i.e., if $d \leq c$, we accept the lot and if $d > c$, we reject the lot.

Under rectifying sampling plan

If $d \leq c$, we accept the lot and replace all defective units found in the sample by non-defective units and if $d > c$, we accept the lot after inspecting the entire lot and replacing all defective units in the lot by non-defective units.

The steps described above are shown in Fig. 7.1.

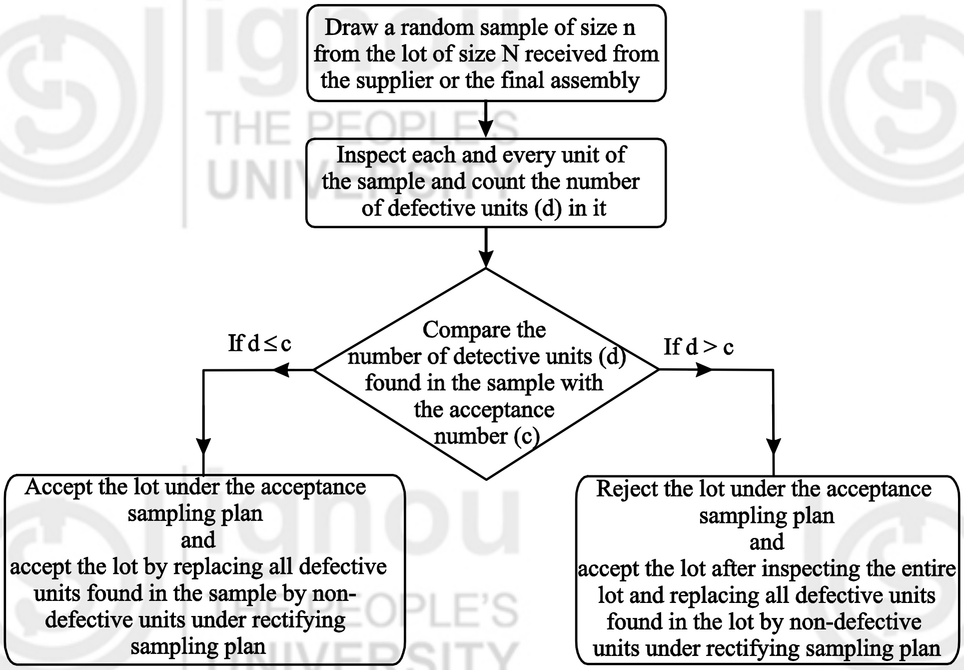

Fig. 7.1: Procedure for implementing a single sampling plan.

Let us explain these steps further with the help of an example.

Example 1: Suppose a mobile phone company produces mobiles phones in lots of 100 phones. To check the quality of the lots, the quality inspector of the company uses a single sampling plan with n = 15 and c = 1. Explain the procedure for implementing it.

Solution: For implementing the single sampling plan, the quality inspector of the company randomly draws a sample of 15 mobile phones from each lot and classifies each mobile of the sample as defective or non-defective. At the end of the inspection, he/she counts the number of defective mobiles (d) found in the sample and compares it with the acceptance number (c). If $d \leq c$ (= 1), he/she accepts the lot and if $d > c$ (= 1), he/she rejects the lot under the acceptance sampling plan. Under rectifying sampling plan, if $d \leq c$ (= 1), he/she accepts the lot by replacing all defective mobiles found in the sample by non-defective mobiles and if $d > c$, he/she accepts the lot by inspecting the entire lot and replacing all defective mobiles in the lot by non-defective mobiles.

You may like to explain the procedure for implementing a single sampling plan yourself. Try the following exercise.

E1) A manufacturer of silicon chip produces lots of 1000 chips for shipment. A buyer uses a single sampling plan with n = 50 and c = 2 to test for bad outgoing lots. Explain the procedure for implementing it under acceptance sampling plan.

So far you have learnt about the single sampling plan and how it is implemented. We describe various features of the single sampling plan in Secs.7.3 to 7.6.

7.3 OPERATING CHARACTERISTIC (OC) CURVE

You have studied in Unit 6 that the operating characteristics (OC) curve is an important aspect of an acceptance sampling plan. This curve displays the discriminatory power of the sampling plan. That is, it shows the probability that a lot submitted with a certain fraction defective will be either accepted or rejected. In this section, we discuss how to construct the OC curve for a single sampling plan.

You have learnt that for constructing an OC curve, we require the probabilities of accepting a lot corresponding to different quality levels. Therefore, we first compute the probability of accepting a lot of incoming quality p for a single sampling plan.

You have studied in Sec. 7.2 that in a single sample plan, we accept the lot if the number of defective units (d) in the sample is less than or equal to the acceptance number (c). It means that if X represents the number of defective units in the sample, we accept the lot if $X \le c$, i.e., X = 0 or 1 or 2, …, or c. Therefore, the probability of accepting the lot of incoming quality p is given by

You have studied in Unit 3 of MST-003 that if A and B are mutually exclusive events then $P[A \text{ or } B] = P[A] + P[B]$

$$P_a(p) = P[X \le c] = P[X = 0 \text{ or } 1, ..., \text{ or } c]$$

$$= P[X=0] + P[X=1] + ... + P[X=c] \quad \left(\because X = 0, 1, 2, ..., c \text{ are mutually exclusive}\right)$$

$$= \sum_{x=0}^{c} P[X = x] \qquad \ldots (1)$$

We can calculate this probability if we know the distribution of X. Generally, in quality control, a random sample is drawn from a lot of finite size without replacement. So in such situations, the number of defective units (X) in the sample follows a hypergeometric distribution. In a lot of size N and incoming quality p, the number of defective units is Np and the number of non-defective units is N – Np. Therefore, the probability of getting exactly x defective units in a sample of size n from this lot is given by

The notation ${}^{Np}C_x$ can also be repersened as $\binom{Np}{x}$.

$$P[X = x] = \frac{{}^{Np}C_x \; {}^{N-Np}C_{n-x}}{{}^{N}C_x}; \; x = 0, 1, ..., \min(Np, n) \qquad \ldots (2)$$

Thus, we can obtain the probability of accepting a lot of quality p in a single sampling plan by putting the value of P[X = x] in equation (1) as follows:

$$P_a(p) = \sum_{x=0}^{c} P[X = x] = \sum_{x=0}^{c} \frac{{}^{Np}C_x \; {}^{N-Np}C_{n-x}}{{}^{N}C_x} \qquad \ldots (3)$$

We know from industrial experience that n is usually small for any economically worthwhile production process. Therefore, when sample size n is small compared to lot size (N), i.e., when $N \ge 10n$, we know that the hypergeometric distribution is approximated by the binomial distribution with parameters n and p where p is the lot quality. It is far easier to calculate the probabilities with the help of the binomial distribution in comparison with the hypergeometric distribution.

Therefore, we can take

$$P[X = x] = {}^{n}C_x \, p^x (1-p)^{n-x}$$

Thus, the probability of accepting a lot of quality p using binomial approximation is given by

$$P_a(p) = \sum_{x=0}^{c} P[X = x] = \sum_{x=0}^{c} {}^nC_x\, p^x (1-p)^{n-x} \qquad \dots (4)$$

However, for rapid calculation, we can use Table I entitled **Cumulative Binomial Probability Distribution** which is given at the end of this block. We can also approximate the binomial distribution to another distribution. When p is small and n is large such that np is finite, we know that the binomial distribution approaches the Poisson distribution with parameter λ = np. Therefore, the probability of accepting a lot of quality p using the Poisson approximation is given by

$$P_a(p) = \sum_{x=0}^{c} \frac{e^{-\lambda}\lambda^x}{\underline{|x}} \qquad \dots (5)$$

For Poisson distribution

$$P[X = x] = \frac{e^{-\lambda}\lambda^x}{\underline{|x}}$$

We can use Table II entitled **Cumulative Poisson Probability Distribution** given at the end of this block for calculating this probability.

Note: Table I and Table II do not list a tabulated value for each value of p and λ, respectively. In such cases, we interpolate it as we have discussed in Unit 4 of MST-004 or use a scientific calculator to calculate the probability of accepting a lot.

We now illustrate how to compute the probability of accepting a lot in different situations with the help of an example.

Example 2: A manufacturer of silicon chip produces chips in lots of 1000. A single sampling plan is used to test for bad outgoing lots. If the quality of incoming lot is 0.01, calculate the probability of accepting the lot in the following cases:

i) n = 12 and c = 1, and

ii) n = 60 and c = 2.

Solution:

i) It is given that

$$N = 1000, n = 12, c = 1 \text{ and } p = 0.01$$

If X represents the number of defective chips in the sample, the lot is accepted if X ≤ c, i.e., X ≤ 1. Therefore, the probability of accepting the lot is given by

$$P_a(p) = P[X \le c] = P[X \le 1] = P[X = 0] + P[X = 1]$$

$$= \sum_{x=0}^{1} P[X = x] = \sum_{x=0}^{1} \frac{{}^{Np}C_x\; {}^{N-Np}C_{n-x}}{{}^NC_x}$$

Since $N \ge 10n$, we can use the binomial distribution (with parameters n and p where p is the lot quality) as the approximation of the hypergeometric distribution. Therefore, the probability of accepting the lot of quality p is given by

$$P_a(p) = P[X \le 1] = \sum_{x=0}^{1} {}^nC_x\, p^x (1-p)^{n-x}$$

For rapid calculation, we can use Table I for obtaining this probability.

From Table I, for n = 12, x = c = 1 and p = 0.01, we have

$$\sum_{x=0}^{1} {}^{n}C_{x}\, p^{x} (1-p)^{n-x} = 0.9938$$

Therefore, the probability of accepting the lot of quality p = 0.01 is given by

$$P_a(p) = P[X \le 1] = \sum_{x=0}^{1} {}^{n}C_{x}\, p^{x} (1-p)^{n-x} = 0.9938$$

ii) It is given that

$$N = 1000, n = 60, c = 2 \text{ and } p = 0.01$$

Since p is small, n is large and np = 0.65 is finite, we can use the Poisson distribution (with parameters λ = np) as the approximation of the hypergeometric distribution. Therefore, the probability of accepting the lot of quality p is given by

$$P_a(p) = P[X \le 2] = \sum_{x=0}^{2} \frac{e^{-\lambda}\lambda^{x}}{\lfloor x}$$

where $\lambda = np = 60 \times 0.01 = 0.60$.

For rapid calculation, we can use Table II for obtaining this probability.

From Table II, for λ = 0. 60 and x = c = 2, we have

$$\sum_{x=0}^{2} \frac{e^{-\lambda}\lambda^{x}}{\lfloor x} = 0.9769$$

Hence, the probability of accepting a lot of quality p = 0.01 is given by

$$P_a(p) = P[X \le 2] = \sum_{x=0}^{2} \frac{e^{-\lambda}\lambda^{x}}{\lfloor x} = 0.9769$$

You can calculate the probability of acceptance of a lot for different lot qualities in the same way.

We now take up the construction of the OC curve for a single sampling plan.

As you know, the OC curve is constructed by taking the quality level (proportion defective) on the X-axis and the probability of accepting the lot on the Y-axis. So for construction of the OC curve, we first consider different quality levels such as p = 0.01, 0.02, 0.03… and then calculate the corresponding probability of acceptaning a lot as discussed in Example 2.

Let us consider an example to demonstrate the construction of the OC curve.

Example 3: Suppose a consumer receives lots of 500 candles from a new supplier. To check the quality of the lot, the consumer draws one sample of size 20 and accepts the lot if the inspected sample contains at most one defective cnadle. Otherwise, he/she rejects the lot. Construct the OC curve for this plan.

Solution: It is given that

$$N = 500, n = 20, c = 1$$

For constructing the OC curve, we have to calculate the probabilities of accepting the lot corresponding to different quality levels.

If X represents the number of defective candles in the sample, the consumer accepts the lot if $X \le c$, i.e., $X \le 1$. Therefore, the probability of accepting the lot is given by

$$P_a(p) = P[X \le c] = P[X \le 1] = \sum_{x=0}^{1} P[X = x]$$

Since $N \ge 10n$, we can use binomial distribution. Therefore,

$$P_a(p) = P[X \le 1] = \sum_{x=0}^{1} {}^nC_x\, p^x (1-p)^{n-x}$$

We use Table I for calculating the probabilities of accepting the lot corresponding to different quality levels such as $p = 0.01, 0.02, 0.03\ldots$ as we have discussed in Example 2. These probabilities are shown in the table given below:

Incoming Lot Quality	**Probability of Accepting the Lot**
0	1
0.01	0.9831
0.02	0.9401
0.04	0.8103
0.06	0.6605
0.08	0.5169
0.10	0.3917
0.12	0.2891
0.14	0.2084
0.16	0.1471
0.18	0.1018
0.20	0.0692

We now construct the OC curve by taking the quality level (proportion defective) on the X-axis and the probability of accepting the lot on the Y-axis as shown in Fig. 7.2.

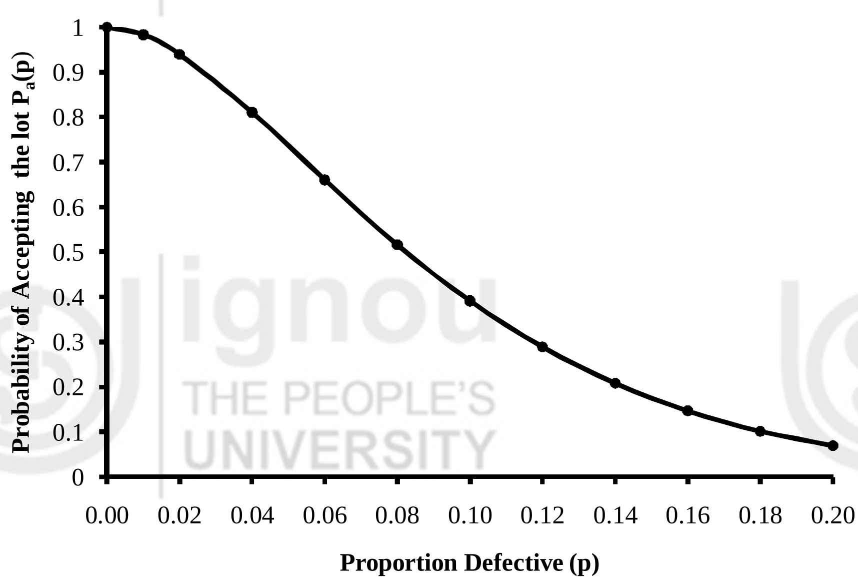

Fig. 7.2: The OC curve for the single sampling plan.

You may now like to construct the OC curve for the following exercise.

E2) A hospital receives disposable injection syringes in lots of 2000. A single sampling plan with n = 25 and c = 2 is being used for inspection by the quality inspector of the hospital. Construct the OC curve for this plan.

You have learnt in Unit 5 that the acceptance or rejection of the entire lot depends on the conclusions drawn from the sample. Thus, there is always a chance of making a wrong decision. It means that a lot of good quality may be rejected and a lot of poor quality may be accepted. This leads to two kinds of risks:

1. Producer's risk, and
2. Consumer's risk.

We now discuss these risks for the single sampling plan.

7.4 PRODUCER'S RISK AND CONSUMER'S RISK

In Unit 5, we have defined the producer's risk as follows:

The probability of rejecting a lot of acceptance quality level (AQL) p_1 is known as the producer's risk.

Therefore, the producer's risk for a single sampling plan is given by

$$P_p = P[\text{rejecting a lot of acceptance quality level } p_1]$$

$$= 1 - P[\text{accepting a lot of acceptance quality level } p_1]$$

$$= 1 - P_a(p_1) \qquad \ldots (6)$$

We can compute $P_a(p_1)$ from equation (3) by replacing the quality level p with p_1 as follows:

$$P_a(p_1) = \sum_{x=0}^{c} P[X = x] = \sum_{x=0}^{c} \frac{{}^{Np_1}C_x \; {}^{N-Np_1}C_{n-x}}{{}^{N}C_x}$$

Therefore, from equation (6), the producer's risk is given by

$$P_p = 1 - P_a(p_1) = 1 - \sum_{x=0}^{c} \frac{{}^{Np_1}C_x \; {}^{N-Np_1}C_{n-x}}{{}^{N}C_x} \qquad \ldots (7)$$

For rapid calculation of the producer's risk for a single sampling plan, we can also use approximations as we have discussed in Sec. 7.4. Therefore, if we use the approximation of the hypergeometric distribution to the binomial distribution with parameters n and p_1, the producer's risk is given by

$$P_p = 1 - P_a(p_1) = 1 - \sum_{x=0}^{c} {}^{n}C_x \, p_1^x \, (1 - p_1)^{n-x} \qquad \ldots (8)$$

We now explain the **Consumer's Risk** for a single sampling plan:

By definition, the probability of accepting a lot of unsatisfactory quality (LTPD) p_2 is known as the consumer's risk.

Therefore, the consumer's risk for a single sampling plan is given by

$$P_c = P[\text{accepting a lot of quality } p_2]$$

We can compute the consumer's risk for the single sampling plan from equation (4) by replacing the quality level p with p_2 as follows:

$$P_c = P_a(p_2) = \sum_{x=0}^{c} P[X = x] = \sum_{x=0}^{c} \frac{{}^{Np_2}C_x \; {}^{N-Np_2}C_{n-x}}{{}^{N}C_x} \quad \ldots (9)$$

If we approximate the hypergeometric distribution to the binomial distribution with parameters n and p_2, the consumer's risk is given by

$$P_c = P_a(p_2) = \sum_{x=0}^{c} {}^{n}C_x \, p_2^x (1-p_2)^{n-x} \quad \ldots (10)$$

Let us take up an example from real life to explain these concepts.

Example 4: Suppose a tyre supplier ships tyres in lots of size 400 to the buyer. A single sampling plan with n = 15 and c = 0 is being used for the lot inspection. The supplier and the buyer's quality control inspector decide that AQL = 0.01 and LTPD = 0.10. Compute the producer's risk and consumer's risk for this single sampling plan.

Solution: It is given that

$$N = 400, n = 15, c = 0, AQL(p_1) = 0.01 \text{ and } LTPD(p_2) = 0.10$$

Since $N \geq 10n$, we can use the binomial distribution. Therefore, we can use equation (8) to calculate the producer's risk for the single sampling plan.

We first calculate the probability of accepting a lot of quality $p = p_1 = 0.01$ using Table I.

From Table I, for n = 15, x = c = 0 and $p = p_1 = 0.01$, we have

$$P_a(p_1) = P[X \leq 0] = \sum_{x=0}^{0} {}^{n}C_x \, p_1^{x} (1-p_1)^{n-x} = 0.8601$$

Therefore, from equation (8), we calculate the producer's risk as follows:

$$P_p = 1 - P_a(p_1) = 1 - 0.8601 = 0.1399$$

It means that if there are several lots of the same quality p = 0.01, about 13.99% of these will be rejected. This is obviously a risk for the supplier because it was agreed upon by both that lots of quality 0.01 will be accepted whereas the quality inspector is rejecting 13.99% of those.

Similarly, we can calculate the consumer's risk using equation (10).

We first calculate the probability of accepting a lot of quality $p = p_2 = 0.10$ using Table I.

From Table I, for n = 15, x = c = 0 and p = 0.10, we have

$$P_a(p_2) = P[X \leq 0] = \sum_{x=0}^{0} {}^{n}C_x \, p_2^x (1-p_2)^{n-x} = 0.2059$$

Therefore, from equation (10), the consumer's risk is given by

$$P_c = P_a(p_2) = 0.2059$$

It means that if there are several lots of the same quality p = 0.10, about 20.59% of these will be accepted by the quality inspector even though this quality is unsatisfactory. This is obviously the buyer's risk.

For practice you can also compute the producer's risk and consumer's risk in the following exercise.

E3) Suppose in E2, the acceptance quality level (AQL) and lot tolerance percent defective (LTPD) are 0.04 and 0.10, respectively. Calculate the producer's risk and consumer's risk for this plan.

7.5 AVERAGE OUTGOING QUALITY (AOQ)

You have studied in Unit 6 that the concept of average outgoing quality is particularly useful in the rectifying sampling plan where the rejected lots are inspected 100% and all defective units are replaced by non-defective units. The AOQ is defined as follows:

The expected quality of the lots after the applications of sampling inspection is called the **average outgoing quality (AOQ)**. It is calculated from the formula given below:

$$AOQ = \frac{\text{Number of defective units in the lot after the inspection}}{\text{Total number of units in the lot}} \quad \ldots (11)$$

So in a single sample plan, we can obtain the formula for average outgoing quality by considering the following situations:

i) If the lot of size N is accepted on the basis of a sample of size n, (N – n) units remain un-inspected. If the incoming quality of the lot is p, we expect that p(N – n) defective units are left in the lot after the inspection. However, the probability that the lot will be accepted by the sampling plan is P_a. Therefore, the expected number of defective units per lot in the outgoing stage is $p(N-n)\,P_a$.

ii) If the lot is rejected, all units of the lot go for 100% inspection and all defective units found in the lot are replaced by non-defective units. So there is no defective unit at the outgoing stage. The probability that the lot will be rejected is $(1 - P_a)$. Therefore, the expected number of defective units per lot at the outgoing stage is $0 \times (1 - P_a) = 0$.

Thus, the expected number of defective units per lot after sampling inspection is $p(N-n)P_a + 0 = p(N-n)P_a$.

Hence, the average proportion defective in the outgoing stage or average outgoing quality (AOQ) is given by

$$AOQ = \frac{\text{Number of defective units in the lot after the inspection}}{\text{Total number of units in the lot}}$$

or $$AOQ = \frac{p(N-n)P_a}{N} \quad \ldots (12)$$

If the sample size n is very small in proportion to the lot size N, i.e., $n/N \approx 0$, equation (12) for AOQ becomes

$$AOQ = p\left(1 - \frac{n}{N}\right)P_a \approx pP_a \quad \ldots (13)$$

Let us now discuss the construction of the AOQ curve for a single sampling plan.

As you know, the AOQ curve is constructed by taking the quality level (proportion defective) on the X-axis and the AOQ on the Y-axis. So for constructing the AOQ curve, we first consider different quality levels such as p = 0.01, 0.02, 0.03… and then calculate the corresponding AOQ.

Let us consider some examples for calculating AOQ and constructing the AOQ curve.

Example 5: Suppose in Example 3 the submitted lot quality is p = 0.02. The rejected lots are screened and all defective candles are replaced by the non-defective candles. Calculate the average outgoing quality (AOQ) for this plan.

Solution: The submitted lot quality is p = 0.02 and we have to calculate the AOQ for this single sampling plan.

It is given that

$$N = 500, n = 20, c = 1 \text{ and } p = 0.02$$

From equation (12), the AOQ for the single sampling plan is

$$AOQ = \frac{p(N-n)P_a}{N}$$

where P_a is the probability of accepting the lot of quality p. Therefore, for calculating AOQ, we have to calculate P_a. We have already calculated this probability in Example 3. So we directly use the result:

$$P_a = 0.9401$$

On putting the values of N = 500, n = 20, p = 0.02 and P_a = 0.9401in equation (12), we get

$$AOQ = \frac{p(N-n)P_a}{N} = \frac{0.02 \times (500-20) \times 0.9401}{500} = 0.018$$

In the same way, you can calculate AOQ for different submitted lot qualities.

Example 6: Suppose in Example 4, the rejected lots are screened and all defective tyres are replaced by non-defective tyres. Construct the AQO curve for this plan.

Solution: It is given that

$$N = 400, n = 15, c = 0$$

For constructing the AOQ curve, we first calculate the probabilities of accepting the lot corresponding to different quality levels using Table I. Then we calculate the AOQ for each quality level using equation (12).

The probabilities of accepting the lot and the AOQs corresponding to different quality levels are given in the following table:

Incoming Lot Quality	Probability of Accepting the Lot	AOQ
0	1	0
0.01	0.8601	0.0083
0.02	0.7386	0.0142

0.04	0.5421	0.0209
0.06	0.3953	0.0228
0.08	0.2863	0.0220
0.10	0.2059	0.0198
0.12	0.1470	0.0170
0.14	0.1041	0.0099
0.16	0.0731	0.0079
0.18	0.0510	0.0061
0.20	0.0352	0.0000

We now construct the AOQ curve by taking the quality level (proportion defective) on the X-axis and the corresponding AOQ values on the Y-axis as shown in Fig.7.3.

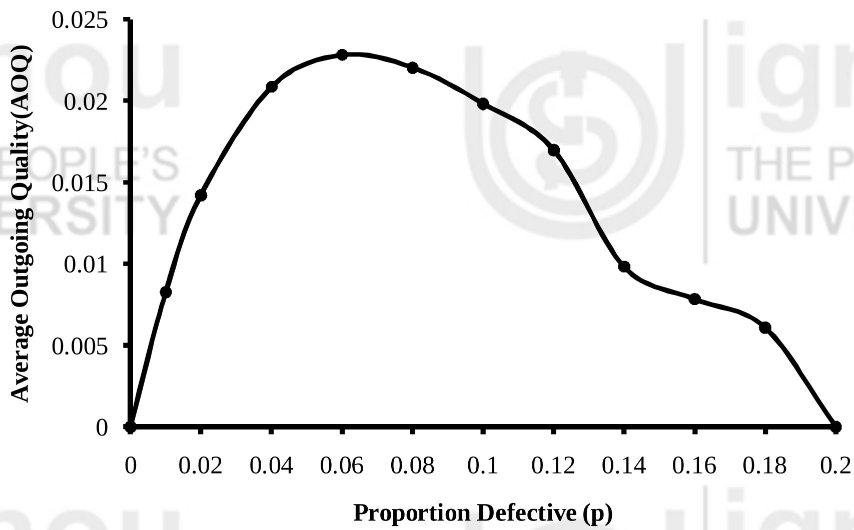

Fig. 7.3: The AOQ curve for Example 5.

You may now like to calculate the AOQ and construct the AOQ curve. Try the following exercises.

E4) Assuming that the lot size is large relative to the sample size, calculate the approximate average outgoing quality (AOQ) for the single sampling plan with n = 10 and c = 0 containing 20% defective units.

E5) A computer manufacturer purchases computer chips from a company in lots of 200. Twelve computer chips are sampled at random and inspected for defects. The computer manufacturer accepts the lot if the inspected sample contains at most one defective chip. Otherwise, he/she rejects the lot. If the rejected lots are screened and all defective computer chips are replaced by non-defective chips. Construct the AOQ curve for this plan.

You have learnt about the OC curve, producer's risk, consumer's risk and AOQ for a single sampling plan. We now discuss ASN and ATI of the plan.

7.6 AVERAGE SAMPLE NUMBER (ASN) AND AVERAGE TOTAL INSPECTION (ATI)

Two other features that are also useful for any sampling plan are the average sample number (ASN) and the average total inspection (ATI). We now discuss these for a single sampling plan in some detail.

Average Sample Number (ASN)

You have studied in Unit 6 that the average sample number is the expected number of sample units per lot, which is required to arrive at a decision about the acceptance or rejection of the lot under the acceptance sampling plan.

In acceptance single sampling plan, the decision about the acceptance or rejection of a lot is taken on the basis of a single sample that has been inspected. Therefore, the ASN in a single sampling plan is simply the sample size n. It means that ASN is constant in a single sampling plan.

Therefore,

$$\text{ASN} = n \qquad \dots (14)$$

The curve drawn between the ASN and the lot quality (p) is known as the **ASN curve**.

The ASN curve for a single sampling plan is a straight line as shown in Fig. 7.4.

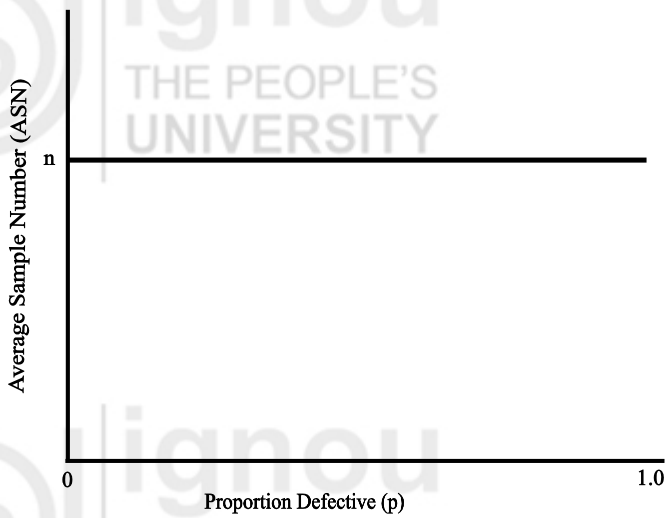

Fig. 7.4: The ASN curve for a single sampling plan.

Let us consider Example 4. In this example, the quality control inspector takes the decision of acceptance or rejection of the lot on the basis of the single sample of size n = 15. So the ASN for this single sampling plan is 15.

Average Total Inspection (ATI)

You know that the concept of average total inspection (ATI) is considered under rectifying sampling plan in which rejected lots under go 100% inspection. It is defined as follows:

The average number of units inspected per lot under the rectifying sampling plan is called the **average total inspection** (ATI).

So in a rectifying single sampling plan, the number of units to be inspected will depend on two situations given below:

i) If the lot of size N is accepted on the basis of a sample of size n, the number of units inspected is n and the probability of the accepting the lot is P_a.

ii) If the lot is rejected on the basis of a sample, we inspect the entire lot of size N and the probability of rejecting the lot is $(1 – P_a)$.

Therefore, we can compute the ATI for a single sampling plan as follows:

ATI = Average number of units inspected per lot

$$= \sum (\text{inspected number of units} \times \text{probability of taking decision})$$

$$ATI = n \times P_a + N \times (1 - P_a)$$

This can also be written as

$$ATI = n + (1 - P_a)(N - n) \quad \ldots (15)$$

The curve drawn between ATI and lot quality (p) is known as the **ATI curve**.

Let us take up an example to illustrate this concept.

Example 7: Calculate the ASN for the plan given in Example 4. If the rejected lots are screened and all defective tyres are replaced by non-defective tyres, construct the ATI curve for this plan.

Solution: It is given that

$$N = 400, n = 15, c = 0$$

In the first case, the decision of acceptance or rejection of the lot is taken only on a single sample of size n = 15. Therefore, the ASN for this plan is simply the sample size, i.e., ASN = n = 15.

For construction of the ATI curve, we first calculate the probabilities of accepting the lot corresponding to different quality levels using Table I. Then we calculate the ATI for each quality level using equation (15).

We have already calculated these probabilities in Example 6 and we use those results. Substituting the values of N, n, p and P_a in equation (15) we can calculate ATI.

The probabilities of accepting the lot and the ATIs corresponding to different quality levels are given in the following table:

Incoming Lot Quality	Probability of Accepting the Lot	ATI
0	1	15.00
0.01	0.8601	68.86
0.02	0.7386	115.64
0.04	0.5421	191.29
0.06	0.3953	247.81
0.08	0.2863	289.77
0.10	0.2059	320.73
0.12	0.1470	343.41
0.14	0.1041	359.92
0.16	0.0731	371.86
0.18	0.0510	380.37
0.20	0.0352	386.45

We now construct the ATI curve by taking the quality level (proportion defective) on the X-axis and the corresponding ATI values on the Y-axis as shown in Fig.7.5.

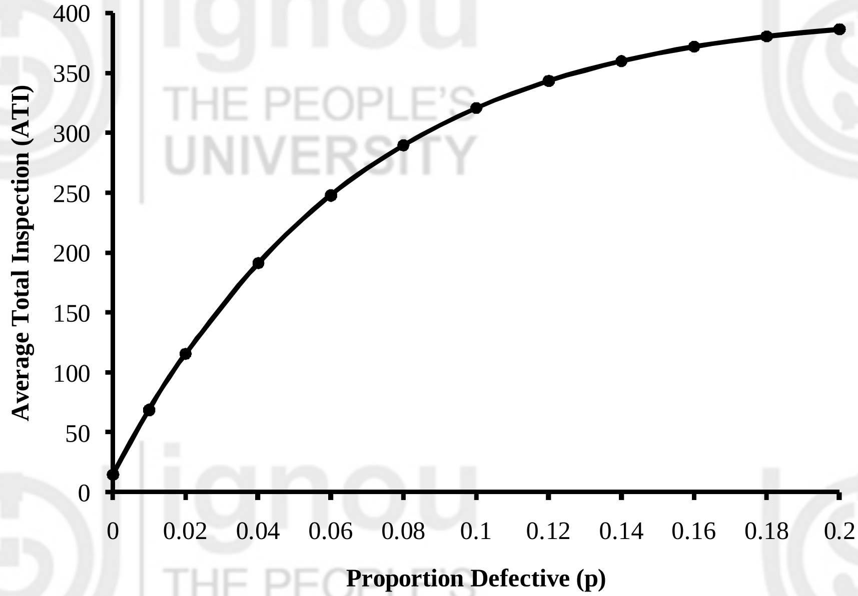

Fig. 7.5: The ATI curve for Example 6.

You can try the following exercise based on ASN and ATI for practice.

E6) Calculate the ASN and construct the ATI curve for the plan given in E5.

So far you have learnt the various features of the single sampling plan. We now discuss how to design a single sampling plan.

7.7 DESIGN OF SINGLE SAMPLING PLANS

The design of a single sampling plan implies the determination of the parameters of the plan, i.e., the sample size **n** and the acceptance number **c**. These numbers have to be decided in advance before applying the single sampling plan technique. There are several approaches for determining the parameters n and c for the single sampling. Sometimes, more than one plan will satisfy the criteria, but the best plan is the one with the largest sample size. It provides adequate protection to both producer and consumer. We now discuss some of the approaches.

7.7.1 Stipulated Producer's Risk

In this approach, the producer's risk α and its corresponding acceptance quality level (AQL) p_1 are specified. We would like to design a single sampling plan in such a way that the lots of quality level p_1 are accepted $100(1-\alpha)\%$ of the time.

According to this approach, we first select an acceptance number (c) and then find the value of np_1 corresponding to c and α with the help of Table III given at the end of this block. Then the value of n is obtained by dividing np_1 by p_1 (= AQL) as follows:

$$n = \frac{np_1}{p_1} \qquad \dots (16)$$

If the computed value of n is a fraction, it is rounded off to the next integer.

For different values of the acceptance number (c), we may get different values of sample size (n) (as you will see in Example 8). This means that for the same producer's risk and acceptance quality level, we have different single sampling plans. If we draw the OC curve for each sampling plan, we will see that each plan has a different consumer's risk. So out of these, we choose the sampling plan which gives the best protection to the consumer against acceptance of poor quality lots.

We now describe this procedure with the help of an example.

Example 8: Suppose a tyre supplier ships tyres in lots of size 400 to the buyer. The supplier and quality control inspector of the buyer decide the acceptance quality level (AQL) to be 2%. Design a sampling plan which ensures that the lots of quality 2% will be rejected 5% of the time.

Solution: The supplier and the quality control inspector desire the sampling plan for which AQL (p_1) = 2% and the producer's risk (α) = 5%.

To find the desired sampling plan, first of all, we choose the acceptance number c as c = 1. Then we look up the value of np_1 corresponding to c = 1 and $\alpha = 0.05$ from Table III. We have

$$np_1 = 0.355$$

Therefore, we can obtain the sample size as follows:

$$n = \frac{np_1}{p_1} = \frac{0.355}{0.02} = 17.75 \approx 18$$

Hence, the required single sampling plan is

$$n = 18 \text{ and } c = 1$$

Similarly, for c = 2 and $\alpha = 0.05$, the value of np_1 is 0.818.

Therefore, the sample size is

$$n = \frac{np_1}{p_1} = \frac{0.818}{0.02} = 40.9 \approx 41$$

Hence, the required single sampling plan is

$$n = 41 \text{ and } c = 2$$

Similarly, for c = 5 and $\alpha = 0.05$, the value of np_1 is 2.613.

Therefore,

$$n = \frac{np_1}{p_1} = \frac{2.613}{0.02} = 130.65 \approx 131$$

Hence, the required single sampling plan is

$$n = 131 \text{ and } c = 5$$

The OC curves for the three sampling plans (as discussed in Sec. 7.4) are shown in Fig. 7.6.

Incoming Lot Quality	Probability of Accepting the Lot		
	n = 18 and c = 1	**n = 41 and c = 2**	**n = 131 and c = 5**
0	1.0000	1.0000	1.0000
0.01	0.9862	0.9920	0.9978
0.02	0.9505	0.9514	0.9513
0.04	0.8393	0.7750	0.5731
0.06	0.7055	0.5505	0.1959

0.08	0.5719	0.3526	0.0445
0.10	0.4503	0.2086	0.0075
0.12	0.3460	0.1156	0.0010
0.14	0.2602	0.0607	0.0001
0.16	0.1920	0.0303	0.0000
0.18	0.1391	0.0145	0.0000
0.20	0.0991	0.0066	0.0000

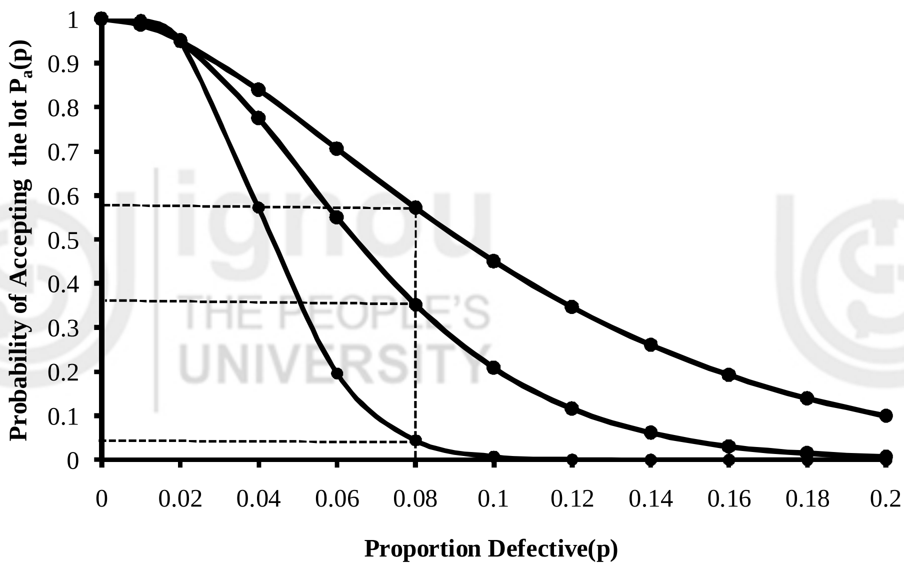

Fig. 7.6: The OC curves for Example 8.

From Fig. 7.6, we conclude that out of the three sampling plans, the sampling plan with n = 131, c = 5 provides the best protection to the consumer because it has the lowest probability of accepting poor quality lots. However, this sampling plan has the largest sample size (n = 131), which increases the inspection cost.

7.7.2 Stipulated Consumer's Risk

According to this approach, the consumer risk β and its corresponding lot tolerance percent defective (LTPD) p_2 are specified. We desire to determine a single sampling plan in such a way that we will be accepted lots of quality level p_2 100β% of the time.

To design the plan using this approach, we first select an acceptance number (c) and then find the value of np_2 corresponding to c and β with the help of Table III. Then the value of n can be obtained by dividing np_2 by p_2 = LTPD as follows:

$$n = \frac{np_2}{p_2} \qquad \dots (17)$$

For different values of c, a number of sampling plans may satisfy this criterion. If we draw the OC curve for each sampling plan, we will see that each plan has a different producer's risk. So out of these, we choose the sampling plan which gives the best protection to the producer against the rejection of good quality lots.

We now describe this procedure with the help of an example.

Example 9: Suppose, in Example 8, the supplier and the quality control inspector decide the lot tolerance percent defective (LTPD) to be 5%. Determine the single sampling plans using c = 1, 2 and 8 which ensure that the lots of quality 5% will be accepted 10% of the time.

Solution: It is given that

LTPD (p_2) = 5% and the consumer'risk (β) = 10%

To find the desired sampling plan, we first look up the value of np_2 corresponding to c = 1 and β = 0.10 from Table III. We have

$$np_2 = 3.890$$

Therefore, the sample size is

$$n = \frac{np_2}{p_2} = \frac{3.890}{0.10} = 38.90 \approx 39$$

Hence, the required single sampling plan is

$$n = 39, c = 1$$

Similarly, for c = 2 and β = 0.10, the value of np_2 is 5.322.

Therefore, the sample size is

$$n = \frac{np_2}{p_2} = \frac{5.322}{0.10} = 53.22 \approx 54$$

Hence, the required single sampling plan is

$$n = 54, c = 2$$

Similarly, for c = 8 and β = 0.10 the value of np_2 is 12.995.

Therefore,

$$n = \frac{np_2}{p_2} = \frac{12.955}{0.10} = 129.55 \approx 130$$

Hence, the required single sampling plan is

$$n = 130, c = 8$$

7.7.3 Stipulated Producer's Risk and Consumer's Risk

According to this approach, the producer's risk with its corresponding acceptance quality level (AQL) and consumer's risk with its corresponding lot tolerance percent defective (LTPD) are specified. We have to design single sampling plans which satisfy both producer's and consumer's risks, such that the lots of AQL are to be rejected no more than 100α% of the time and lots of LTPD are to be accepted no more than 100β% of the time.

Here the criteria are more stringent than the previous approaches and we may not have much flexibility in choosing the acceptance number and the associated sampling plan. So it may be difficult to design a sampling plan that will satisfy both producer's and consumer's risks.

In this approach, we first find the operating ratio R as follows:

$$R = \frac{p_2}{p_1} \qquad \dots (18)$$

The values of R corresponding to various acceptance number (c) and α and β are also listed in Table III. We choose a value of R from Table III which is exactly equal to the desired value of R corresponding to the desired α and β. Generally, the tabulated value of R is not equal to the desired value of R. So in such situations, we take the tabulated values of R between which the desired value of R lies. Then we look up the corresponding values of acceptance number (c) in Table III and find the value of sample size (n). The following two approaches are used to find n:

1. **Satisfy Producer's Risk Stipulation Exactly and come Close to Consumer's Risk**

   According to this approach, we find the values of n as we have discussed in Sec. 7.7.1. It means that we first find the value of np_1 corresponding to c and α with the help of Table III. Then we find n from equation (16). Thus,

   $$n = \frac{np_1}{p_1}$$

   The values of n are obtained for both values of c. In this way, we get two sampling plans which satisfy producer's risk exactly.

   Out of these, we choose the sampling plan which is close to satisfying the consumer's risk. For that we find the value of p_2 for each plan. We find the values of np_2 corresponding to the desired β and each c from Table III. Then we find the value of p_2 for each plan as follows:

   $$p_2 = \frac{np_2}{n} \qquad \dots (19)$$

   We choose the sampling plan for which the calculated p_2 is closer to the desired p_2.

   Another criterion for choosing the sampling plan is that we select the sampling plan which has the smallest sample size in order to minimize inspection costs. Alternatively, we can choose the sampling plan which has the largest sample size in order to get maximum information.

2. **Satisfy Consumer's Risk Stipulation Exactly and come Close to Producer's Risk**

   According to this approach, we find the values of n as we have discussed in Sec. 7.7.2. It means that we first find the value of np_2 corresponding to c and β with the help of Table III. Then we find n from equation (17). Thus,

   $$n = \frac{np_2}{p_2}$$

   The values of n are obtained for both values of c. In this way, we get two sampling plans which satisfy consumer's risk exactly.

   Out of these, we choose the sampling plan which is close to satisfying the producer's risk. For that we find the value of p_1 for each plan. We find the values of np_1 corresponding to the desired α and each c from Table III. Then we find the value of p_1 for each plan as follows:

   $$p_1 = \frac{np_1}{n} \qquad \dots (20)$$

We choose the sampling plan for which the calculated p_1 is closer to the desired p_1.

Another criterion for choosing the sampling plan is that we select the sampling plan which has the smallest sample size in order to minimize inspection costs. Alternatively, we can choose the sampling plan which has the largest sample size in order to get maximum information.

We now describe this procedure with the help of an example.

Example 10: Suppose, in Example 8, the supplier and the quality control inspector decide the acceptance quality level (AQL) to be 2% and the lot tolerance percent defective (LTPD) to be 8%. Design a sampling plan which ensures that lots of quality 2% will be rejected 5% of the time and lots of quality 8% will be accepted 5% of the time.

Solution: It is given that

$$\text{AQL} = p_1 = 2\% = 0.02 \text{ and } \alpha = 5\% = 0.05$$

$$\text{LTPD} = p_2 = 8\% = 0.08 \text{ and } \beta = 5\% = 0.05$$

To design the desired sampling plan, we first calculate the operating ratio (R) from equation (18) as follows:

$$R = \frac{p_2}{p_1} = \frac{0.08}{0.02} = 4.0$$

From Table III, we see that the desired value of R = 4.0 lies between 4.023 and 3.604 for $\alpha = 0.05$ and $\beta = 0.05$. From Table III, the corresponding acceptance numbers (c) are 5 and 6. We have to find the value of sample size (n).

We first find the plans which satisfy the desired producer's risk exactly.

For this, we find the value of np_1 corresponding to c = 5 and $\alpha = 0.05$ with the help of Table III.

From Table III, for c = 5 and $\alpha = 0.05$, we have $np_1 = 2.613$. Therefore, from equation (16), we have

$$n = \frac{np_1}{p_1} = \frac{2.613}{0.02} = 130.65 \approx 131$$

Similarly, for c = 6 and $\alpha = 0.05$, the value of np_1 is 3.286. Therefore,

$$n = \frac{np_1}{p_1} = \frac{3.286}{0.02} = 164.3 \approx 165$$

So both the plans with n = 131, c = 5 and n = 165, c = 6 satisfy the producer's risk exactly. Out of these plans, we have to find the plan which is closer to satisfying the desired consumer's risk. For this we find np_2 corresponding to c and β.

From Table III, for c = 5, $\beta = 0.05$, we have $np_2 = 10.531$.

Therefore, from equation (19), we have

$$p_2 = \frac{np_2}{n} = \frac{10.513}{131} = 0.08$$

Similarly, for c = 6, $\beta = 0.05$, we have $np_2 = 11.842$.

Therefore,

$$p_2 = \frac{np_2}{n} = \frac{11.842}{165} = 0.07$$

Since the value of $p_2 = 0.08$ corresponding to the plan n = 131, c = 5 is equal to the desired value 0.08, the plan with n = 131, c = 5 is the best single sampling plan.

We now find the plans which satisfy the desired consumer's risk exactly.

From Table III, for c = 5 and $\beta = 0.05$, we have $np_2 = 10.513$. Therefore, from equation (17), we have

$$n = \frac{np_2}{p_2} = \frac{10.513}{0.08} = 131.4 \approx 132$$

Similarly, for c = 6 and $\beta = 0.05$, we have $np_2 = 11.842$. Therefore,

$$n = \frac{np_2}{p_2} = \frac{11.842}{0.08} = 148.02 \approx 149$$

So both the plans with n = 132, c = 5 and n = 149, c = 6 are satisfied the consumer's risk exactly. Out of these plans, we have to find the plan which is closer to the desired producer's risk. For this we find np_1 corresponding to c and α.

From Table III, for c = 5, $\alpha = 0.05$, we have $np_1 = 2.613$

Therefore,

$$p_1 = \frac{np_1}{n} = \frac{2.613}{132} = 0.0198$$

Similarly, for c = 6, $\alpha = 0.05$, we have $np_1 = 3.286$.

Therefore,

$$p_1 = \frac{np_1}{n} = \frac{3.286}{149} = 0.022$$

Since the value of $p_1 = 0.0198$ corresponding to the plan n = 132, c = 5 is approximate equal to the desired value 0.02, the plan n = 132, c = 5 is the best single sampling plan.

7.7.4 Larson Binomial Nomograph

There is also a graphical method for designing the single sampling plans when the producer's risk with corresponding acceptance quality level (AQL) and consumer's risk with corresponding lot tolerance percent defective (LTPD) are specified.

This method is based on Larson binomial nomograph and is used when we take the binomial approximate to the hypergeometric distribution, i.e., when $N \geq 10n$. The Larson binomial nomograph (shown in Fig. 7.7) is a graph of the cumulative binomial distribution. It has two scales. On the left scale, **proportion defective (p)** is shown as **probability of occurences on a single trial (p)**. This scale is known as **p-scale**. On the right scale, the **probability of acceptance (P_a)** is shown as **probability of less than or equal to c occurences in n trials (p)**. This scale is known as **P_a-scale**.

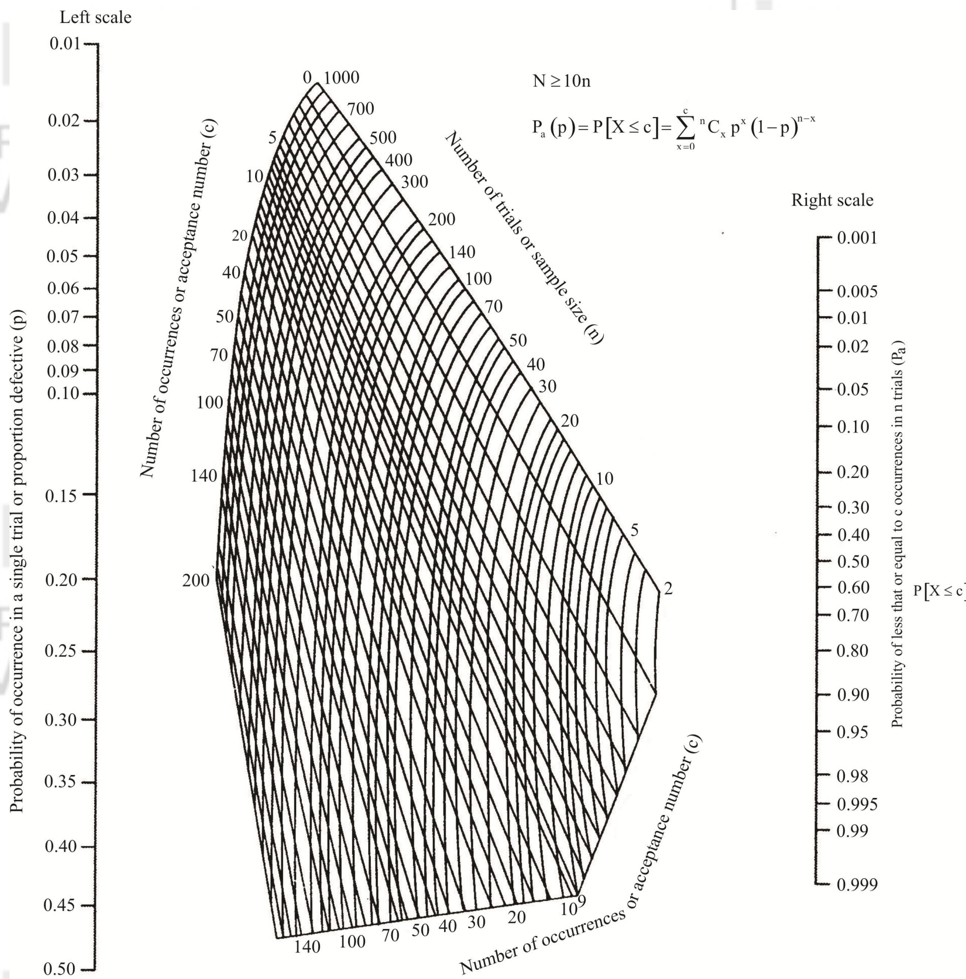

Fig. 7.7: Larson binomial nomograph.

The procedure of designing a single sampling plan by using the nomograph is quite simple. We plot the values of AQL (p_1) and LTPD (p_2) on the left scale and the corresponding values of $(1 - \alpha)$ and β on the right scale. Then we join p_1 with $(1 - \alpha)$ and p_2 with β by straight lines. We read the values of sample size (n) and acceptance number (c) at the intersection of the lines on the grid.

To demonstrate this method, we consider Example 10.

It is given that

$$\text{AQL} = p_1 = 2\% = 0.02 \text{ and } \alpha = 5\% = 0.05$$

$$\text{LTPD} = p_2 = 8\% = 0.08 \text{ and } \beta = 5\% = 0.05$$

For designing a single sampling plan by using Larson nomograph, we first plot $p_1 = 0.02$ and $p_2 = 0.08$ on the p-scale on the nomograph. Then we plot $1 - \alpha$ (= 0.95) and $\beta = 0.05$ on the P_a-scale on the nomograph. After plotting these points, we draw a straight line joining p_1 (= 0.02) and $1 - \alpha$ (= 0.95) and another straight line joining p_2 (= 0.08) and β (= 0.05) as shown in Fig. 7.8. At the intersection of the two lines, we read the values of n and c.

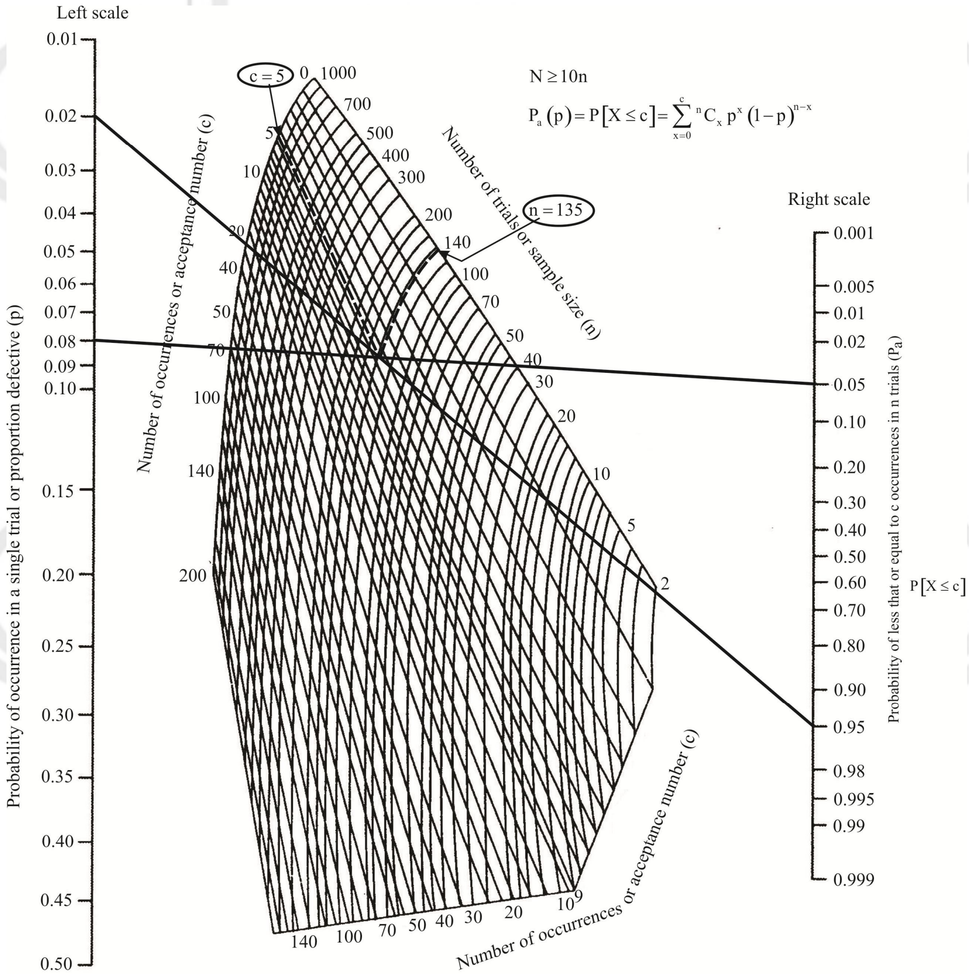

Fig. 7.8

From Fig. 7.8 we have, n = 135 and c = 5.

You can now check your understanding of how to design a single sampling plan by answering the following exercises.

E7) A ball bearing supplier and an automobile company have decided to check the quality of ball bearings in lots of size 1000 with acceptance quality level (AQL) as 1%. Design the single sampling plans using c = 2, 4 and 6 which ensure that lots of quality 1% will be rejected 1% of the time.

E8) A consumer receives lots of 5000 candles from a new supplier. To check the quality of lots, the consumer and supplier want to use the single sampling plan which satisfies a consumer's risk of 5% for lots of quality 5% . Determine sampling plans for the specified consumer's risk and LTPD for acceptance number c = 3 and 6.

We end this unit by giving a summary of what we have covered in it.

7.8 SUMMARY

1. The main acceptance sampling plans for attributes are:
   i) Single sampling plan,
   ii) Double sampling plan,

iii) Multiple sampling plan, and

iv) Sequential Sampling Plan.

2. A sampling plan in which a decision about the acceptance or rejection of a lot is based on a single sample that has been inspected is known as a **single sampling plan**. There are two parameters of a single sampling plan:

   n – size of the sample, and

   c – acceptance number for the sample.

3. In a single sampling plan, if number of defective units (d) in the sample is less than or equal to the stated acceptance number (c), i.e., if $d \le c$, we accept the lot and if $d > c$, we reject the lot under acceptance sampling plan.

4. In a single sampling plan, if $d \le c$, we accept the lot and replace all defective units found in the sample by non-defective units and if $d > c$, we accept the lot by inspecting the entire lot and replacing all defective units in the lot by non-defective units under rectifying sampling plan.

5. The probability of accepting a lot of quality p for a single sampling plan is given by

$$P_a(p) = P[X \le c] = \sum_{x=0}^{c} {}^{n}C_x\, p^x (1-p)^{n-x}$$

6. The produce's risk and consumer's risk for a single sampling plan are given by

$$P_p = 1 - P_a(p_1) = 1 - \sum_{x=0}^{c} {}^{n}C_x\, p_1^x (1-p_x)^{n-x} \text{ and}$$

$$P_c = P_a(p_2) = \sum_{x=0}^{c} {}^{n}C_x\, p_2^x (1-p_2)^{n-x}$$

7. The AOQ for a single sampling plan is

$$AOQ = \frac{p(N-n)P_a}{N}$$

8. The ASN and ATI for a single sampling plan are

$$ASN = n \text{ and } ATI = n + (1-P_a)(N-n)$$

9. Designing a single sampling plan implies determining the sample size (n) and acceptance number (c).

7.9 SOLUTIONS/ANSWERS

E1) To check the quality of the lots, the buyer randomly draws 50 silicon chips from each lot. After that he/she inspects each and every chip drawn from the lot for certain defects and classifies each chip of the sample as defective or non-defective. At the end of the inspection, he/she counts the number of defective chips (d) found in the sample and then compares the number of defective chips (d) with the acceptance number (c). If $d \le c = 2$, he/she accepts the lot and if $d > c = 2$, he/she rejects the lot on the basis of the inspected sample. It means that if the buyer finds 0 or 1 or 2 defective chips in the sample, he/she accepts the lot. Otherwise, he/she rejects the lot.

E2) It is given that

$$N = 2000, n = 25, c = 2$$

For constructing the OC curve, we have to calculate the probabilities of accepting the lot corresponding to different quality levels.

If X represents the number of defective syringes in the sample, the quality inspector accepts the lot if $X \le c = 2$. Therefore, the probability of accepting the lot is given by

$$P_a(p) = P[X \le c] = P[X \le 2] = \sum_{x=0}^{2} P[X = x]$$

Since $N \ge 10n$, we use the binomial distribution. We can use Table I for calculating the probabilities of accepting the lot corresponding to different quality levels such as $p = 0.01, 0.02, 0.03 \ldots$ .These probabilities are shown in the table given below:

Incoming Lot Quality	**Probability of Accepting the Lot**
0	1
0.01	0.9980
0.02	0.9868
0.04	0.9235
0.06	0.8129
0.08	0.6768
0.10	0.5371
0.12	0.4088
0.14	0.3000
0.16	0.2130
0.18	0.1467
0.20	0.0982

We construct the OC curve by taking the quality level (proportion defective) on the X-axis and the probability of accepting the lot on the Y-axis as shown in Fig. 7.9.

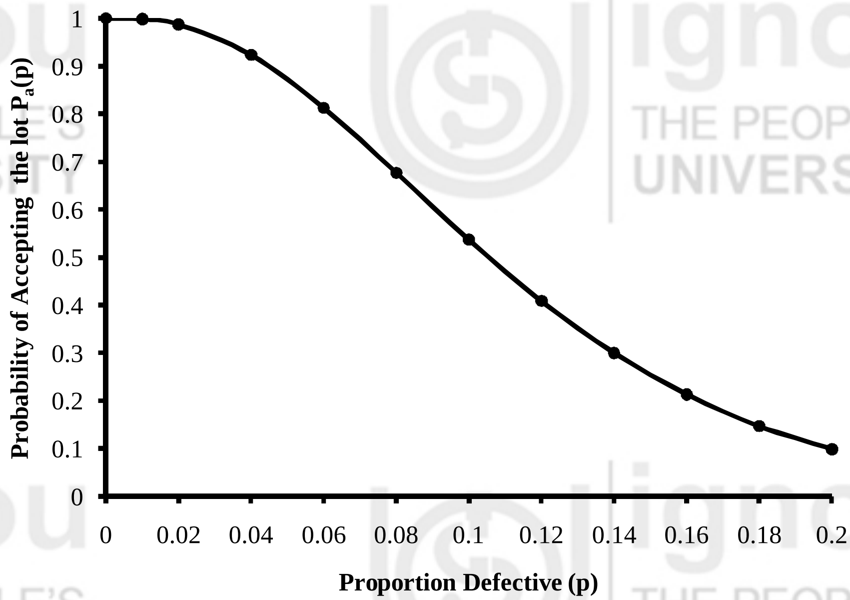

Fig. 7.9: The OC curve for E2.

E3) It is given that

$$N = 2000, n = 25, c = 2, AOL(p_1) = 0.04 \text{ and } LTPD(p_2) = 0.10$$

Since $N \geq 10n$, we use the binomial distribution. Therefore, we use equation (8) to calculate the producer's risk for the single sampling plan.

We first calculate the probability of accepting the lot of quality $p = p_1 = AQL = 0.04$ using Table I.

From Table I, for n = 25, x = c = 2 and $p = p_1 = 0.04$, we have

$$P_a(p_1) = P[X \leq 2] = \sum_{x=0}^{2} {}^nC_x\, p_1^x (1-p_1)^{n-x} = 0.9235$$

Therefore, from equation (8), we have

$$P_p = 1 - P_a(p_1) = 1 - 0.9235 = 0.0765$$

It means that if there are several lots of the same quality p = 0.04, about 7.65% of these will be rejected. This is obviously a risk for the supplier because it was agreed upon by both that lots of quality 0.04 will be accepted whereas the quality inspector is rejecting 7.65% of them.

Similarly, we can calculate the consumer's risk using equation (10).

We first calculate the probability of accepting the lot of quality $p = p_2 = LTPD = 0.10$ using Table I.

From Table I, for n = 25, x = c = 2 and $p = p_2 = 0.10$, we have

$$P_a(p_2) = P[X \leq 2] = \sum_{x=0}^{2} {}^nC_x\, p_2^x (1-p_2)^{n-x} = 0.5371$$

Therefore, from equation (10), we have the consumer's risk

$$P_c = P_a(p_2) = 0.5371$$

It means that if there are several lots of the same quality p = 0.10, about 53.71% out of these will be accepted by the quality inspector even though this quality is unsatisfactory. This is obviously a risk for the quality inspector.

E4) It is given that

$$n = 10, c = 0, p = 20\% = 0.20$$

Since the lot size is large relative to the sample size, we can calculate the average outgoing quality (AOQ) for the single sampling plan using equation (13).

For calculating AOQ, we have to calculate the probability of accepting the lot corresponding to p = 0.20.

If X represents the number of defective units in the sample, the lot is accepted if $X \leq c = 0$. Therefore, the probability of accepting the lot is given by

$$P_a(p) = P_a = P[X \leq 0]$$

Since the lot size is large relative to the sample size, we can calculate this probability by using Table I.

From Table I, for n = 10, x = c = 0 and p = 0.20, we have

$$P_a = P[X \leq 0] = \sum_{x=0}^{0} {}^nC_x \, p^x (1-p)^{n-x} = 0.1074$$

On putting the values of p and P_a in equation (13), we get

$$AOQ = pP_a = 0.20 \times 0.1074 = 0.0215 = 2.15\%$$

E5) It is given that

$$N = 200, n = 12, c = 1$$

Since $N \geq 10n$, we can use the binomial distribution. Therefore, we use equation (12) to calculate the AOQ.

For constructing the AOQ curve, we have to calculate the probabilities of accepting the lot corresponding to different quality levels.

If X represents the number of defective chips in the sample, the manufacturer accepts the lot if $X \leq c = 1$. Therefore, the probability of accepting the lot is given by

$$P_a(p) = P[X \leq 1] = \sum_{x=0}^{1} P[X = x]$$

We can use Table I for calculating the probabilities of accepting the lot corresponding to different quality levels such as p = 0.01, 0.02, 0.03… Then we calculate AOQ for each quality level by using equation (12). The probabilities of accepting the lot and AOQs corresponding to different quality levels are given in the following table:

Incoming Lot Quality	Probabilty of Accepting the Lot	AOQ
0	1.0000	0
0.01	0.9938	0.0093
0.02	0.9769	0.0184

0.04	0.9191	0.0346
0.06	0.8405	0.0474
0.08	0.7513	0.0565
0.10	0.6590	0.0619
0.12	0.5686	0.0641
0.14	0.4834	0.0636
0.16	0.4055	0.0610
0.18	0.3359	0.0568
0.20	0.2749	0.0517

We construct the AOQ curve by taking the quality level (proportion defective) on the X-axis and the corresponding AOQ values on the Y-axis as shown in Fig.7.10.

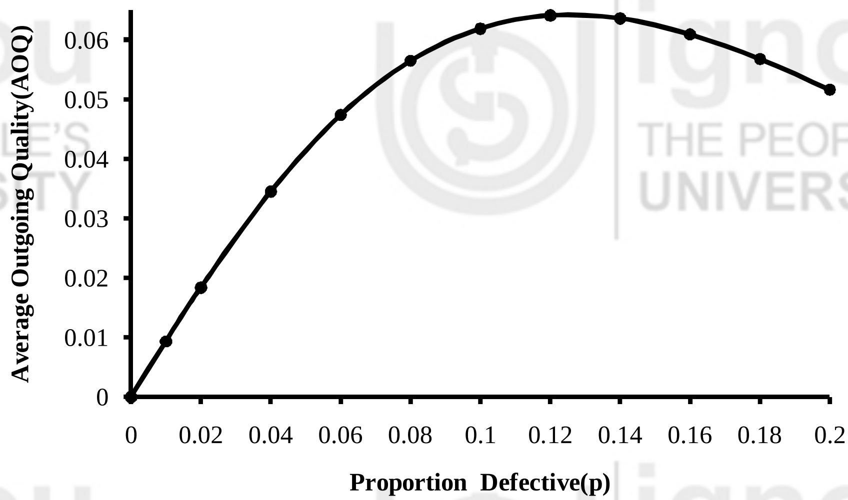

Fig. 7.10: The AOQ curve for E5.

E6) It is given that

$$N = 200, n = 12, c = 1$$

In the first case, the decision of acceptance or rejection of the lot is taken only on a single sample of size n = 12. Therefore, the ASN for this plan is simply the sample size, i.e., ASN = n = 12.

For construction of the ATI curve, we first calculate the probabilities of accepting the lot corresponding to different quality levels using Table I. Then we calculate the ATI for each quality level using equation (15).

We have already calculated these probabilities in E5 and we those results. Substituting the values of N, n, p and P_a in equation (15) we can calculate ATI.

The probabilities of accepting the lot and the ATIs corresponding to different quality levels are given in the following table:

Incoming Lot Quality	Probability of Accepting the Lot	ATI
0	1.0000	12.00
0.01	0.9938	13.17
0.02	0.9769	16.34
0.04	0.9191	27.21
0.06	0.8405	41.99

0.08	0.7513	58.76
0.10	0.6590	76.11
0.12	0.5686	93.10
0.14	0.4834	109.12
0.16	0.4055	123.77
0.18	0.3359	136.85
0.20	0.2749	148.32

We now construct the ATI curve by taking the quality level (proportion defective) on the X-axis and the corresponding ATI values on the Y-axis as shown in Fig.7.11.

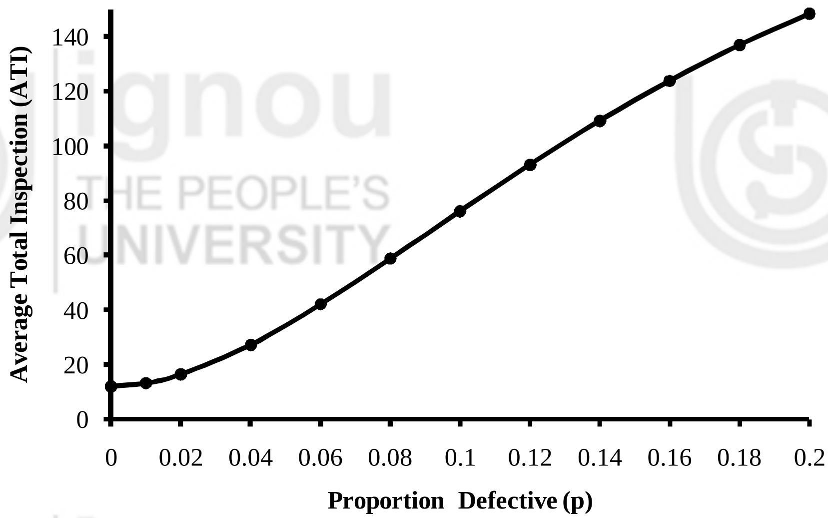

Fig. 7.11: The ATI curve for E6.

E7) We have

$$AQL = 1\% = 0.01 \text{ and } \alpha = 1\% = 0.01$$

To design the desired sampling plan, we first look up the value of np_1 corresponding to $c = 2$ and $\alpha = 0.01$ from Table III. We have

$$np_1 = 0.436$$

Therefore, the sample size is

$$n = \frac{np_1}{p_1} = \frac{0.436}{0.01} = 43.6 \approx 44$$

Hence, the required single sampling plan is

$$n = 44 \text{ and } c = 2$$

Similarly, for $c = 4$ and $\alpha = 0.01$, the value of np_1 is 1.279.

Therefore, the sample size is

$$n = \frac{np_1}{p_1} = \frac{1.279}{0.01} = 127.9 \approx 128$$

Hence, the required single sampling plan is

$$n = 128 \text{ and } c = 4$$

Similarly, for $c = 6$ and $\alpha = 0.01$, the value of np_1 is 2.330.

Therefore,

$$n = \frac{np_1}{p_1} = \frac{2.330}{0.01} = 233$$

Hence, the required single sampling plan is

$$n = 233 \text{ and } c = 6$$

E8) We have

$$LTPD = p_2 = 5\% = 0.05 \text{ and } \beta = 5\% = 0.05$$

To design the desired sampling plan, we look up the value of np_2 corresponding to $c = 3$ and $\beta = 0.05$ from Table III. We have

$$np_2 = 7.754$$

Therefore, the sample size is

$$n = \frac{np_2}{p_2} = \frac{7.754}{0.05} = 155.08 \approx 156$$

Hence, the required single sampling plan is

$$n = 156 \text{ and } c = 3$$

Similarly, for $c = 6$ and $\beta = 0.05$, the value of np_2 is 11.842.

Therefore, the sample size is

$$n = \frac{np_2}{p_2} = \frac{11.842}{0.05} = 236.84 \approx 237$$

Hence, the required single sampling plan is

$$n = 237 \text{ and } c = 6$$

UNIT 8 DOUBLE SAMPLING PLANS

Structure

8.1 INTRODUCTION

In Unit 7, you have learnt about the single sampling plan and the procedure for implementing it. In a single sampling plan, we take the decision of accepting or rejecting a lot on the basis of a single sample.

Sometimes, situations arise when it is not possible to decide whether to accept or reject the lot on the basis of a single sample. In such situations, we use a sampling plan known as the **double sampling plan**. In this plan, the decision of acceptance or rejection of a lot is taken on the basis of **two samples**. A lot may be accepted immediately if the first sample is good or may be rejected if it is bad. If the first sample is neither good nor bad, the decision is based on the evidence of the first and second sample combined.

In this unit, we explain the concept of the double sampling plan and the procedure for implementing it (Sec. 8.2). In Secs. 8.3 to 8.7, we discuss various features of the double sampling plan viz. the OC curve, producer's risk and consumer's risk, AOQ, ASN and ATI. In the last section of the unit (Sec. 8.8), we explain how to design the double sampling plan. In the next unit, we shall introduce decision theory.

Objectives

After studying this unit, you should be able to:

- describe a double sampling plan and explain how to implement it;
- differentiate between single and double sampling plans;
- compute the probability of accepting or rejecting a lot of incoming quality in a double sampling plan;
- construct the operating characteristic (OC) curve for a double sampling plan;

- compute the producer's risk and consumer's risk for a double sampling plan;
- compute the average sample number (ASN) and the average total inspection (ATI) for a double sampling plan; and
- design double sampling plans.

8.2 DOUBLE SAMPLING PLAN

A sampling plan in which a decision about the acceptance or rejection of a lot is based on two samples that have been inspected is known as a **double sampling plan**.

The double sampling plan is used when a clear decision about acceptance or rejection of a lot cannot be taken on the basis of a single sample. In double sampling plan, generally, the decision of acceptance or rejection of a lot is taken on the basis of two samples. If the first sample is bad, the lot may be rejected on the first sample and a second sample need not be drawn. If the first sample is good, the lot may be accepted on the first sample and a second sample is not needed. But if the first sample is neither good nor bad and there is a doubt about its results, we take a second sample and the decision of acceptance or rejection of a lot is taken on the basis of the evidence obtained from both the first and the second samples.

For example, suppose a buyer purchases cricket balls in lots of 500 from a company. To check the quality of the lots, the buyer and the company decide that the buyer will draw two samples of sizes 10 (first sample) and 20 (second sample) and the acceptance numbers for the plan are 1 and 3. The buyer takes two samples and makes the decision of acceptance or rejection of the lot on the basis of two samples. Since the decision of acceptance or rejection of the lot is taken on the basis of two samples, this is a double sampling plan.

A double sampling plan requires the specification of four quantities which are known as its **parameters**. These parameters are

n_1 – size of the first sample,

c_1 – acceptance number for the first sample,

n_2 – size of the second sample, and

c_2 – acceptance numbers for both samples combined.

Therefore, the parameters of the double sampling plan in the above example are

the size of the first sample (n_1) = 10,

the acceptance number for the first sample (c_1), = 1

the size of the second sample (n_2) = 20, and

the acceptance numbers for both the samples combined (c_2) = 3.

So far you have learnt the definition of the double sampling plan and why it is used. We now describe the procedure for implementing it and its advantages over the single sampling plan.

8.2.1 Implementation of Double Sampling Plan

Suppose, lots of the same size, say N, are received from the supplier or the final assembly line and submitted for inspection one at a time. The procedure for implementing the double sampling plan to arrive at a decision about the lot is described in the following steps:

Step 1: We draw a random sample (first sample) of size n_1 from the lot received from the supplier or the final assembly.

Step 2: We inspect each and every unit of the sample and classify it as defective or non-defective. At the end of the inspection, we count the number of defective units found in the sample. Suppose the number of defective units found in the first sample is d_1.

Step 3: We compare the number of defective units (d_1) found in the first sample with the stated acceptance numbers c_1 and c_2.

Step 4: We take the decision on the basis of the first sample as follows:

Under acceptance sampling plan

If the number of defective units (d_1) in the first sample is less than or equal to the stated acceptance number (c_1) for the first sample, i.e., if $d_1 \le c_1$, we accept the lot and if $d_1 > c_2$, we reject the lot. But if $c_1 < d_1 \le c_2$, the first (single) sample is failed.

Under rectifying sampling plan

If $d_1 \le c_1$, we accept the lot and replace all defective units found in the sample by non-defective units. If $d_1 > c_2$, we accept the lot after inspecting the entire lot and replacing all defective units in the lot by non-defective units. But if $c_1 < d_1 \le c_2$, the first (single) sample is failed.

Step 5: If $c_1 < d_1 \le c_2$, we draw a second random sample of size n_2 from the lot.

Step 6: We inspect each and every unit of the second sample and count the number of defective units found in it. Suppose the number of defective units found in the second sample is d_2.

Step 7: We combine the number of defective units (d_1 and d_2) found in both samples and consider $d_1 + d_2$ for taking the decision about the lot on the basis of the second sample as follows:

Under acceptance sampling plan

If $d_1 + d_2 \le c_2$, we accept the lot and if $d_1 + d_2 > c_2$, we reject the lot.

Under rectifying sampling plan

If $d_1 + d_2 \le c_2$, we accept the lot and replace all defective units found in the second sample by non-defective units. If $d_1 + d_2 > c_2$, we accept the lot after inspecting the entire lot and replacing all defective units in the lot by non-defective units.

The steps described above are shown in Fig. 8.1 for the acceptance sampling plan.

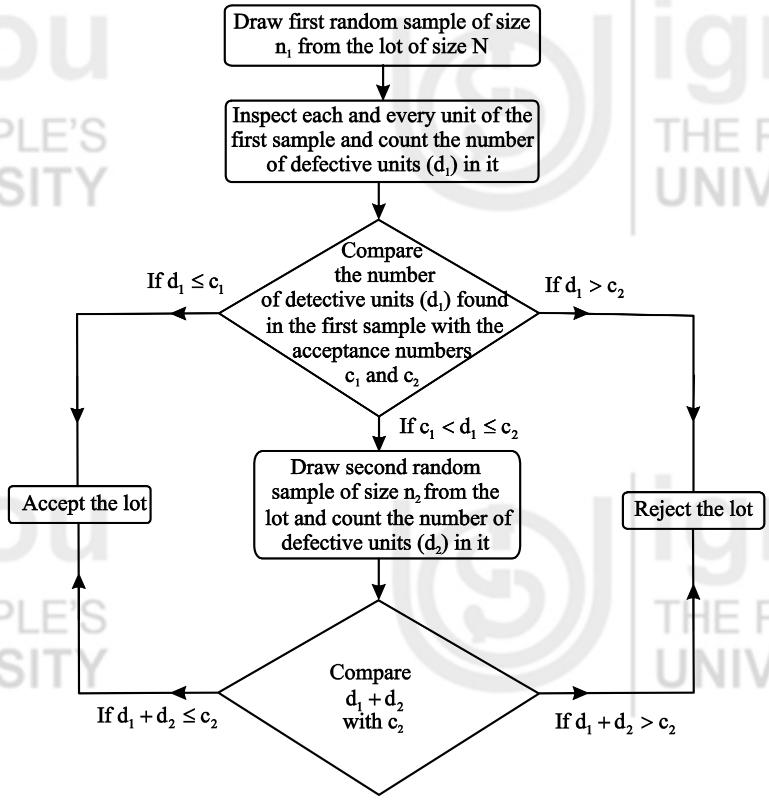

Fig. 8.1: Procedure for implementation of an acceptance double sampling plan.

Let us explain these steps further with the help of an example.

Example 1: Suppose a mobile phone company produces mobile phones in lots of 400 phones each. To check the quality of the lots, the quality inspector of the company uses a double sampling plan with $n_1 = 15$, $c_1 = 1$, $n_2 = 30$, $c_2 = 3$. Explain the procedure for implementing it under acceptance sampling plan.

Solution: For implementing the double sampling plan, the quality inspector of the company randomly draws first sample of 15 mobiles from the lot and classifies each mobile of the first sample as defective or non-defective. At the end of the inspection, he/she counts the number of defective mobiles (d_1) found in the first sample and compares d_1 with the acceptance numbers c_1 and c_2. If $d_1 \le c_1 = 1$, he/she accepts the lot and if $d_1 > c_2 = 3$, he/she rejects the lot. If $c_1 < d_1 \le c_2$, it means that if the number of defective mobiles is 2 or 3, he /she draws the second sample from the lot. He/she counts the number of defective mobiles (d_2) found in the second sample and compares the total number of defective mobiles ($d_1 + d_2$) in both samples with the acceptance number c_2. If $d_1 + d_2 \le c_2 = 3$, he/she accepts the lot and if $d_1 + d_2 > c_2 = 3$, he/she rejects the lot.

8.2.2 Advantages of Double Sampling Plan

A double sampling plan has the following two main advantages over a single sampling plan:

i) The principal advantage of the double sampling plan over the single sample plan is that for the same degree of protection (i.e., the same probability of accepting a lot of a given quality), the double sampling plan may have a smaller average sample number (ASN) than that corresponding to the single sampling plan. The underlying reason is that the size (n_1) of the first sample in the double sampling plan is always smaller than the sample size

(n) of an equivalent single sampling plan. Thus, if a decision is taken on the basis of the first sample, ASN will be lower for the double sampling plan or if a decision is taken after the second sample, the ASN will be reduced.

ii) The double sampling plan has the psychological advantage of giving a lot a second chance. From the viewpoint of the producer/manufacture, it is unfair to reject a lot on the basis of a single sample. The double sampling plan permits the decision to be made on the basis of two samples.

However, double sampling plans are costlier to administer in comparison with the single sampling plans.

Try the following exercise to explain the procedure for implementing a double sampling plan.

E1) A manufacturer of silicon chip produces lots of 1000 chips for shipment. A buyer uses a double sampling plan with $n_1 = 5$, $c_1 = 0$, $n_2 = 20$, $c_2 = 2$ to test the quality of the lots. Explain the procedure for implementing it under acceptance sampling plan.

8.2.3 Difference between Single and Double Sampling Plans

The differences between single and double sampling plans are given below:

1. Single sampling plans are easier to design and operate in comparison with the double sampling plans.
2. The advantage of double sampling plans over single sampling plans seems to be psychological. It looks unreasonable to reject a lot on the basis of one sample and it is more convincing to say that the lot was rejected on the basis of inspection of two samples.
3. Under the double sampling plan, the good lot will generally be accepted and the bad lot will usually be rejected on the basis of the first sample. Thus, in all cases, where a decision to accept or reject is taken on the basis of the first sample, there is a considerable saving in the amount of inspection than required by a comparable single sampling plan. Moreover, when a second sample is taken, it may be possible to reject the lot without completely inspecting the entire second sample (see Sec.8.6).
4. In the double sampling plan, on an average, 25% to 33 % less number of items/units need to be inspected as compared to the single sampling plan.
5. The operating characteristic (OC) curve for a double sampling plan is steeper than a single sampling plan, i.e., the discriminatory power of the double sampling plan is higher than that of the single sampling plan.

So far you have learnt what a double sampling plan is and how it is implemented in industry. You have also learnt the differences between the double and single sampling plans. In Secs. 8.3 to 8.7, we describe various features of the double sampling plan.

8.3 OPERATING CHARACTERISTIC (OC) CURVE

You have learnt in Unit 6 that the operating characteristic (OC) curve displays the discriminatory power of the sampling plan. That is, it shows the probability that a lot submitted with a certain fraction defective will either be accepted or rejected.

You have learnt in Units 6 and 7 that for constructing an OC curve, we require the probabilities of accepting a lot corresponding to different quality levels. Therefore, we now describe how to compute the probability of accepting a lot in a double sampling plan.

You have learnt in Sec. 8.2.1 that in a double sampling plan, the decision of acceptance or rejection of the lot is taken on the basis of two samples. The lot is accepted on the first sample if the number of defective units (d_1) in the first sample is less than the acceptance number c_1. The lot is accepted on the second sample if the number of defective units ($d_1 + d_2$) in both samples is greater than c_1 and less than or equal to the acceptance number c_2. Therefore, if $P_{a1}(p)$ and $P_{a2}(p)$ denote the probabilities of accepting a lot on the first sample and the second sample, respectively, the probability of accepting a lot of quality level p is given by:

$$P_a(p) = P_{a1}(p) + P_{a2}(p) \qquad \ldots (1)$$

If X and Y denote the observed number of defective units in the first and second samples, respectively, we accept the lot on the first sample if $X \le c_1$, i.e., if X = 0 or 1or 2,…, or c_1.

You have studied in Unit 3 of MST-003 that if A and B are mutually exclusive events then
$P[A \text{ or } B] = P[A] + P[B]$

Therefore,

$$P_{a1}(p) = P[X \le c_1] = P[X = 0 \text{ or } 1 \text{ or } 2, ..., \text{or } c_1]$$

$$= P[X = 0] + P[X = 1] + ... + P[X = c_1] \quad \left(\because X = 0, 1, 2, ..., c_1 \text{ are mutually exclusive}\right)$$

$$= \sum_{x=0}^{c_1} P[X = x] \qquad \ldots (2)$$

We can calculate this probability if we know the distribution of X. You have learnt in Unit 5 that generally, in quality control, a random sample is drawn from a lot of finite size without replacement. So in such situations, the number of defective units (X) in the sample follows a hypergeometric distribution.

In a lot of size N and incoming quality p, the number of defective units is Np and non-defective units is N – Np. Therefore, we can compute the probability of getting exactly x defective units in a sample of size n_1 using the hypergeometric distribution as follows:

$$P[X = x] = \frac{{}^{Np}C_x \, {}^{N-Np}C_{n_1 - x}}{{}^{N}C_{n_1}}; \; x = 0, 1, ..., \min(Np, n_1) \qquad \ldots (3)$$

Thus, we can obtain the probability of accepting a lot of quality p on the first sample by putting the value of P[X = x] in equation (2) as follows:

$$P_{a1}(p) = \sum_{x=0}^{c_1} P[X = x] = \sum_{x=0}^{c_1} \frac{{}^{Np}C_x \, {}^{N-Np}C_{n_1 - x}}{{}^{N}C_{n_1}} \qquad \ldots (4)$$

We accept the lot of quality p on the second sample if $c_1 < X + Y \le c_2$. It means that we accept the lot if

$X = c_1 + 1$ and $Y \le c_2 - X = c_2 - c_1 - 1$

or $X = c_1 + 2$ and $Y \le c_2 - c_1 - 2$

$\vdots$

or $X = c_2$ and $Y \le c_2 - c_2 = 0.$

Therefore, from the addition theorem of probability, we can obtain the probability of accepting a lot of quality p on the second sample as follows:

$$P_{a2}(p) = P\left[X = c_1 + 1\right]P\left[Y \le c_2 - c_1 - 1\right] + P\left[X = c_1 + 2\right]P\left[Y \le c_2 - c_1 - 2\right] + \ldots + P\left[X = c_2\right]P\left[Y \le 0\right]$$

$$= \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} P\left[X = x\right]P\left[Y = y / X = x\right] \qquad \ldots (5)$$

where $P\left[Y = y / X = x\right]$ is the conditional probability of observing y defective units in the second sample under the condition that x defective units have already appeared in the first sample.

Since it is given that x defective units have already appeared in the first sample, the remaining defective units and non-defective units in the lot on the second sample are Np – x and $(N - n_1) - (NP - x)$, respectively. Thus, we can compute the probability $P\left[Y = y / X = x\right]$ by using the hypergeometric distribution as follows:

$$P(Y = y / X = x) = \frac{{}^{Np-x}C_y \; {}^{N-n_1-(Np-x)}C_{n_2-y}}{{}^{N-n_1}C_{n_2}} \qquad \ldots (6)$$

On putting the values of $P\left[X = x\right]$ and $P\left[Y = y / X = x\right]$ from equations (4) and (6) in equation (5), we get

$$P_{a2}(p) = \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} \frac{{}^{Np}C_x \; {}^{N-Np}C_{n_1-x}}{{}^{N}C_{n_1}} \frac{{}^{Np-x}C_y \; {}^{N-n_1-(Np-x)}C_{n_2-y}}{{}^{N-n_1}C_{n_2}} \qquad \ldots (7)$$

Therefore, the required probability of accepting a lot of quality p in a double sampling plan is given by

$$P_a(p) = \sum_{x=0}^{c_1} \frac{{}^{Np}C_x \; {}^{N-Np}C_{n_1-x}}{{}^{N}C_{n_1}} + \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} \frac{{}^{Np}C_x \; {}^{N-Np}C_{n_1-x}}{{}^{N}C_{n_1}} \frac{{}^{Np-x}C_y \; {}^{N-n_1-(Np-x)}C_{n_2-y}}{{}^{N-n_1}C_{n_2}} \qquad \ldots (8)$$

However, we can approximate the hypergeometric distribution to the binomial distribution with parameters n and p as we have explained in Unit 7. It means that if the sample sizes n_1 and n_2 are small compared to the lot size (N), the probability of accepting a lot of quality p (using the binomial approximation) is given by

$$P_a(p) = \sum_{x=0}^{c_1} {}^{n_1}C_x \, p^x (1-p)^{n_1-x} + \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} {}^{n_1}C_x \, p^x (1-p)^{n_1-x} \; {}^{n_2}C_y \, p^y (1-p)^{n_2-y} \qquad \ldots (9)$$

However, for rapid calculation, we can use Table I entitled **Cumulative Binomial Probability Distribution** given at the end of this block for calculating this probability.

When p is small and n is large such that np is finite, we know that the binomial distribution approaches the Poisson distribution with parameter $\lambda = np$. Therefore, the probability of accepting a lot of quality p using the Poisson approximation is given by

$$P_a(p) = \sum_{x=0}^{c_1} \frac{e^{-\lambda_1}\lambda_1^x}{\lfloor x} + \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} \frac{e^{-\lambda_1}\lambda_1^x}{\lfloor x} \frac{e^{-\lambda_2}\lambda_2^y}{\lfloor y} \qquad \ldots (10)$$

where $\lambda_1 = n_1 p$ and $\lambda_2 = n_2 p$.

We can use Table II entitled **Cumulative Poisson Probability Distribution** given at the end of this block for calculating this probability.

We now illustrate how to compute the probability of accepting a lot in a double sampling plan.

Example 2: Suppose, in Example 1, the incoming quality of the lot is 0.05. What is the probability of accepting the lot on the first sample? What is the probability of final acceptance?

Solution: It is given that

$$N = 400, n_1 = 15, c_1 = 1, n_2 = 30, c_2 = 3 \text{ and } p = 0.05$$

If $P_{a1}(p)$ and $P_{a2}(p)$ denote the probabilities of accepting the lot on the first sample and the second sample, respectively, we can calculate the probability of accepting the lot of quality level (p) in a double sampling plan as follows:

$$P_a(p) = P_{a1}(p) + P_{a2}(p) \qquad \ldots (i)$$

If X represents the number of defective mobiles in the first sample of size 15, the lot is accepted on the first sample if $X \le c_1 = 1$. Therefore,

$$P_{a1}(p) = P[X \le c_1] = P[X \le 1] \qquad \ldots (ii)$$

Since $N \ge 10n_1$, we can use the binomial distribution and calculate $P_{a1}(p)$ using Table I.

From Table I, for $n = n_1 = 15, x = c_1 = 1$ and $p = 0.05$, we have

$$P_{a1}(p) = P[X \le 1] = \sum_{x=0}^{1} {}^nC_x\, p^x (1-p)^{n-x} = 0.8290 \qquad \ldots (iii)$$

Hence, the probability of accepting the lot on the first sample is 0.8290. It means that about 82.9% of the lot of quality p = 0.05 will be accepted on the first sample.

If Y represents the number of defective mobiles in the second sample, the lot is accepted on the second sample if $c_1 < X + Y \le c_2$, i.e., $1 < X + Y \le 3$. It means that we shall accept the lot if

$X = c_1 + 1 = 1 + 1 = 2$ and $Y \le c_2 - X = 3 - 2 = 1$

or $X = c_1 + 2 = 1 + 2 = 3$ and $Y \le c_2 - X = 3 - 3 = 0$.

Therefore,

$$P_{a2}(p) = P[X=2]P[Y\le 1] + P[X=3]P[Y\le 0] \quad \ldots \text{(iv)}$$

The probabilities $P[X=2]$ and $P[X=1]$ can also be obtained using Table I, which contains the cumulative probabilities of the binomial distribution, using the following expression:

$$P[X=x] = P[X\le x] - P[X\le x-1] \quad \ldots \text{(v)}$$

From Table I, for $n = n_1 = 15$ and $p = 0.05$, we have

$$P[X=2] = P[X\le 2] - P[X\le 1] = 0.9638 - 0.8290 = 0.1348$$

$$P[X=1] = P[X\le 1] - P[X\le 0] = 0.8290 - 0.4633 = 0.3657$$

From Table I, for $n = n_2 = 30$ and $p = 0.05$, we have

$$P[Y\le 1] = 0.5535 \text{ and } P[Y\le 0] = 0.2146$$

On putting these values in equation (iv), we get

$$P_{a2}(p) = P[X=2]P[Y\le 1] + P[X=3]P[Y\le 0]$$

$$= 0.1348\times 0.5535 + 0.3657\times 0.2146 = 0.1531$$

Hence, the probability of accepting the lot on the second sample is 0.1531. It means that about 15.31% of the lot of quality p = 0.05 will be accepted on the second sample.

From equation (i), we get the probability of accepting the lot in the double sampling plan as follows:

$$P_a(p) = P_{a1}(p) + P_{a2}(p) = 0.8290 + 0.1531 = 0.9821$$

Thus, overall, 98.21% of the lots will be accepted by this sampling plan.

In a similar way, we can calculate the probability of accepting a lot for different lot qualities.

The construction of the OC curve for a double sampling plan is beyond the scope of this course. If you are interested in constructing the OC curve, you may consider different quality levels such as p = 0.01, 0.02, 0.03… and then calculate the corresponding probabilities of accepting the lot as explained in Example 2. The OC curve may then be constructed by taking the quality level (proportion defective) on the X-axis and the probability of accepting the lot on the Y-axis as explained for the single sampling plan in Unit 7.

You may now like to calculate the probability of accepting a lot for practice. Try the following exercise.

E2) Suppose that in E1, the incoming quality of the lot is 0.03. Calculate the probabilities of accepting the lot on the first sample and on the second sample. What is the probability of accepting the lot?

8.4 PRODUCER'S RISK AND CONSUMER'S RISK

In Units 5 and 7, we have defined the **producer's risk** as follows:

The probability of rejecting a lot of acceptance quality level (p_1) is known as the producer's risk.

Thus, the producer's risk for a double sampling plan is given by

$$P_p = P\left[\text{rejecting a lot of acceptance quality level } p_1\right]$$
$$= 1 - P\left[\text{accepting a lot of acceptance quality level } p_1\right]$$
$$= 1 - P_a\left(p_1\right) \qquad \dots (11)$$

We can compute $P_a\left(p_1\right)$ from equation (8) by replacing the quality level p with p_1 as follows:

$$P_a(p_1) = \sum_{x=0}^{c_1} \frac{{}^{Np_1}C_x \; {}^{N-Np_1}C_{n_1-x}}{{}^{N}C_{n_1}} + \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} \frac{{}^{Np_1}C_x \; {}^{N-Np_1}C_{n_1-x}}{{}^{N}C_{n_1}} . \frac{{}^{Np_1-x}C_y \; {}^{N-n_1-(Np_1-x)}C_{n_2-y}}{{}^{N-n_1}C_{n_2}} \qquad \dots (12)$$

Therefore, from equation (11), the producer's risk for a double sampling plan is given by

$$P_p = 1 - \sum_{x=0}^{c_1} \frac{{}^{Np_1}C_x \; {}^{N-Np_1}C_{n_1-x}}{{}^{N}C_{n_1}} - \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} \frac{{}^{Np_1}C_x \; {}^{N-Np_1}C_{n_1-x}}{{}^{N}C_{n_1}} . \frac{{}^{Np_1-x}C_y \; {}^{N-n_1-(Np_1-x)}C_{n_2-y}}{{}^{N-n_1}C_{n_2}} \qquad \dots (13)$$

For rapid calculation of the producer's risk for a double sampling plan, we can also use the approximations as explained in Sec. 8.3. Therefore, if we use the approximation of the hypergeometric distribution to the binomial distribution, the producer's risk is given by

$$P_p = 1 - \sum_{x=0}^{c_1} {}^{n_1}C_x \, p_1^x \left(1-p_1\right)^{n_1-x} - \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} {}^{n_1}C_x \, p_1^x \left(1-p_1\right)^{n_1-x} \, {}^{n_2}C_y \, p_1^y \left(1-p_1\right)^{n_2-y} \qquad \dots (14)$$

We now explain the **Consumer's Risk** for a double sampling plan:

By definition, the probability of accepting a lot of unsatisfactory quality (LTPD) p_2 is known as the consumer's risk.

Therefore, the consumer's risk for a double sampling plan is given by

$$P_c = P\left[\text{accepting a lot of quality (LTPD) } p_2\right] = P_a\left(p_2\right)$$

We can compute the consumer's risk for a double sampling plan from equation (8) by replacing the quality level p with the quality LTPD (p_2) as follows:

$$P_c = P_a(p_2) = \sum_{x=0}^{c_1} \frac{{}^{Np_2}C_x \; {}^{N-Np_2}C_{n_1-x}}{{}^{N}C_{n_1}} + \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} \frac{{}^{Np_2}C_x \; {}^{N-Np_2}C_{n_1-x}}{{}^{N}C_{n_1}} . \frac{{}^{Np_2-x}C_y \; {}^{N-n_2-(Np_2-x)}C_{n_2-y}}{{}^{N-n_1}C_{n_2}} \qquad \dots (15)$$

If we approximate the hypergeometric distribution to the binomial distribution, the consumer's risk is given by

$$P_c = P_a\left(p_2\right) = \sum_{x=0}^{c_1} {}^{n_1}C_x\, p_2^x \left(1-p_2\right)^{n_1-x}$$
$$+ \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} {}^{n_1}C_x\, p_2^x \left(1-p_2\right)^{n_1-x}\; {}^{n_2}C_y\, p_2^y \left(1-p_2\right)^{n_2-y} \qquad \text{... (16)}$$

Let us consider an example from a real life situation to explain these concepts.

Example 3: Suppose a shirt manufacturing company supplies shirts in lots of size 500 to the buyer. A double sampling plan with $n_1 = 10$, $c_1 = 0$, $n_2 = 25$, $c_2 = 1$ is being used for the lot inspection. The company and the buyer's quality control inspector decide that AQL = 0.04 and LTPD = 0.10. Compute the producer's risk and consumer's risk for this sampling plan.

Solution: It is given that

$$N = 500, n_1 = 10, c_1 = 0, n_2 = 25, c_2 = 1,$$

$$\text{AQL}(p_1) = 0.04 \text{ and LTPD}(p_2) = 0.10$$

We know that the producer's risk for a double sampling plan is given by

$$P_p = P\left[\text{rejecting a lot of acceptance quality level } p_1\right]$$

$$= 1 - P\left[\text{accepting a lot of acceptance quality level } p_1\right]$$

$$= 1 - P_a\left(p_1\right) \qquad \text{… (i)}$$

Therefore, for calculating the producer's risk, we first calculate the probability of accepting the lot of quality $p = p_1$ = AQL = 0.04 as we have explained in Sec. 8.3.

In a double sampling plan, the lot may be accepted either on the first sample or on the second sample. So if $P_{a1}(p)$ and $P_{a2}(p)$ denote the probabilities of accepting the lot on the first sample and on the second sample, respectively, we can calculate the probability of accepting the lot of quality level p, as follows:

$$P_a(p) = P_{a1}(p) + P_{a2}(p) \qquad \text{… (ii)}$$

If X represents the number of defective shirts in the first sample, the lot will be accepted on the first sample if $X \le c_1 = 0$. Therefore,

$$P_{a1}\left(p\right) = P\left[X \le c\right] = P\left[X \le 0\right] \qquad \text{… (iii)}$$

Since $N \ge 10n$, we can use the binomial distribution and calculate the probability using Table I.

From Table I, for $n = n_1 = 10, x = c_1 = 0$ and $p = p_1 = 0.04$, we have

$$P_{a1}\left(p_1\right) = P\left[X \le 0\right] = \sum_{x=0}^{0} {}^{n_1}C_x\, p_1^x \left(1-p_1\right)^{n_1-x} = 0.6648 \qquad \text{… (iv)}$$

If Y represents the number of defective shirts in the second sample, the lot will be accepted on the second sample if $c_1 < X + Y \le c_2$. It means that we shall accept the lot if $X = c_1 + 1 = 0 + 1 = 1$ and $Y \le c_2 - X = 1 - 1 = 0$.

Therefore,

$$P_{a2}(p_1) = P[X=1]P[Y \le 0] \qquad \ldots (v)$$

We also know that $P[X = x] = P[X \le x] - P[X \le x-1]$

From Table I, for $n = n_1 = 10$ and $p = p_1 = 0.04,$ we have

$$P[X=1] = P[X \le 1] - P[X \le 0] = 0.9418 - 0.6648 = 0.2770$$

From Table I, for $n = n_2 = 25, x = y = 0$ and $p = p_1 = 0.04,$ we have

$$P[Y \le 0] = 0.3604$$

On putting these values in equation (v), we get

$$P_{a2}(p_1) = P[X=1]P[Y \le 0]$$

$$= 0.2770 \times 0.3604 = 0.0998$$

Hence, from equation (ii), we get the probability of accepting the lot in this double sampling plan as follows:

$$P_a(p_1) = P_{a1}(p_1) + P_{a2}(p_1) = 0.6648 + 0.0998 = 0.7646$$

Therefore, we calculate the producer's risk for this plan using equation (i) as follows:

$$P_p = 1 - P_a(p_1) = 1 - 0.7646 = 0.2354$$

It means that if there are several lots of the same quality p = 0.04, about 23.54% out of these will be rejected. This is obviously a risk for the manufacturing company because it was agreed upon by both that lots of quality 0.04 will be accepted whereas the quality inspector is rejecting 23.54% of them.

Similarly, we can calculate the consumer's risk as follows:

We know the consumer's risk for a double sampling plan is given by

$$P_c = P[\text{accepting a lot of quality (LTPD)}\, p_2] = P_a(p_2) \qquad \ldots (vi)$$

We first calculate the probability of accepting the lot of quality $p = p_2 = \text{LTPD} = 0.10$ using equations (ii), (iii) and (v).

From Table I, for $n = n_1 = 10, x = c_1 = 0$ and $p = p_2 = 0.10,$ we have

$$P_{a1}(p_2) = P[X \le 0] = \sum_{x=0}^{0} {}^{n_1}C_x\, p_2^x (1-p_2)^{n-x} = 0.3487 \qquad \ldots (vii)$$

From Table I, for $n = n_1 = 10$ and $p = p_2 = 0.10,$ we have

$$P[X=1] = P[X \le 1] - P[X \le 0] = 0.7361 - 0.3487 = 0.3874$$

From Table I, for $n = n_2 = 25, x = y = 0$ and $p = p_2 = 0.10,$ we have

$$P[Y \le 0] = 0.0718$$

On putting these values in equation (v), we get

$$P_{a2}(p_2) = P[X=1]P[Y \le 0]$$

$$= 0.3874 \times 0.0718 = 0.0278 \qquad \ldots (viii)$$

On putting the values of $P_{a1}(p_2)$ and $P_{a2}(p_2)$ in equation (ii), we get

$$P_a(p_2) = 0.3487 + 0.0278 = 0.3765$$

Hence, from equation (vi), we get the consumer's risk for this plan as follows:

$$P_c = P_a(p_2) = 0.3765$$

It means that if there are several lots of the same quality p = 0.10, about 37.65% out of these will be accepted by the quality inspector even though this quality is unsatisfactory. This is obviously the buyer's risk.

For practice, you can also compute the producer's risk and consumer's risk in the following exercise.

E3) Suppose that in E2, the acceptance quality level (AQL) and lot tolerance percent defective (LTPD) are 0.05 and 0.14, respectively. Calculate the producer's risk and consumer's risk for this plan.

8.5 AVERAGE OUTGOING QUALITY (AOQ)

In Unit 6, you have learnt that the average outgoing quality (AOQ) is defined as follows:

The expected quality of the lots after the application of sampling inspection is called the **average outgoing quality (AOQ)**. It is given by:

$$AOQ = \frac{\text{Number of defective units in the lot after the inspection}}{\text{Total number of units in the lot}} \quad \ldots (17)$$

You know that the concept of AOQ is particularly useful in the rectifying sampling plan wherein the rejected lots are inspected 100% and all defective units are replaced by non-defective units. So in a double sample plan, we can obtain the formula for average outgoing quality by considering the following situations:

i) If the lot of size N is accepted on the first sample of size n_1, $(N - n_1)$ units remain un-inspected. If the incoming quality of the lot is p, we expect that $p(N - n_1)$ defective units are left in the lot after the inspection on the first sample. The probability that the lot will be accepted on the first sample is P_{a1}. Therefore, the expected number of defective units per lot in the outgoing stage is $p(N - n_1)P_{a1}$.

ii) If the lot is rejected on the first sample, all units of the lot go for 100% inspection and all defective units found in the lot are replaced by non-defective units. So there is no defective unit in the outgoing stage. However, the probability that the lot will be rejected on the first sample is $(1 - P_{a1})$. Therefore, the expected number of defective units per lot at the outgoing stage is $0 \times (1 - P_{a1}) = 0$.

iii) If the lot is accepted on the second sample of size n_2, $(N - n_1 - n_2)$ units remain un-inspected. If the incoming quality of the lot is p, we expect that $p(N - n_1 - n_2)$ defective units are left in the lot after the inspection on the second sample. The probability that the lot will be accepted on the second sample is P_{a2}. Therefore, the expected number of defective units per lot in the outgoing stage is $p\left(N-n_1-n_2\right)P_{a2}$.

iv) If the lot is rejected on the second sample, all units of the lot go for 100% inspection and all defective units found in the lot are replaced by non-defective units. So there is no defective unit in the outgoing stage. However, the probability that the lot will be rejected on the second sample is $(1-P_{a2})$. Therefore, the expected number of defective units per lot at the outgoing stage is $0\times(1-P_{a2})=0$.

Thus, the expected number of defective units per lot after sampling inspection is $p\left(N-n_1\right)P_{a1}+0+p\left(N-n_1-n_2\right)P_{a2}+0=p\left[\left(N-n_1\right)\left(P_{a1}+P_{a2}\right)-n_2P_{a2}\right]$.

Hence, from equation (17), the AOQ for a double sampling plan is given by

$$AOQ=\frac{\text{Number of defective units in the lot after the inspection}}{\text{Total number of units in the lot}}$$

$$=\frac{p\left[\left(N-n_1\right)\left(P_{a1}+P_{a2}\right)-n_2P_{a2}\right]}{N}$$

or $$AOQ=p\left[\left(1-\frac{n_1}{N}\right)\left(P_{a1}+P_{a2}\right)-\frac{n_2}{N}P_{a2}\right] \qquad \ldots (18)$$

If the sample sizes n_1 and n_2 are very small in proportion to the lot size N, i.e., $n_1/N \to 0$ and $n_2/N \to 0$, equation (18) for AOQ becomes

$$AOQ=p\left(P_{a1}+P_{a2}\right) \qquad \ldots (19)$$

Let us consider an example to illustrate how to calculate the AOQ for a double sampling plan.

Example 4: Suppose that in Example 2, the rejected lots are screened and all defective mobiles are replaced by non-defective ones. Calculate the average outgoing quality (AOQ) for this plan.

Solution: It is given that

$$N=400, n_1=15, c_1=1, n_2=30, c_2=3 \text{ and } p=0.05$$

Since $N>10n_1$ and $N>10n_2$, we can calculate the average outgoing quality (AOQ) for the double sampling plan using equation (18).

For calculating AOQ, we have to calculate the probabilities of accepting the lot on the first sample and on the second sample corresponding to p = 0.05.

We have already calculated these probabilities in Example 2. So we directly use the results:

$$P_{a1}=0.8290 \text{ and } P_{a2}=0.1531$$

Substituting the values of N, n_1, n_2, P_{a1}, P_{a2} and p in equation (18), we get

$$AOQ=p\left[\left(1-\frac{n_1}{N}\right)\left(P_{a1}+P_{a2}\right)-\frac{n_2}{N}P_{a2}\right]$$

$$=0.05\left[\left(1-\frac{15}{400}\right)(0.8290+0.1531)-\frac{30}{400}\times0.1531\right]$$

$$= 0.05[0.9625 \times 0.9821 - 0.0115] = 0.0467$$

In the same way, you can calculate the AOQ for different lot qualities.

The construction of the AOQ curve for a double sampling plan is beyond the scope of this course. If you are interested in constructing the AOQ curve, you may consider different quality levels such as p = 0.01, 0.02, 0.03… and then calculate the corresponding probabilities of accepting the lot. The AOQ can then be constructed as explained in Units 6 and 7.

You may now like to calculate the average outgoing quality (AOQ) in the following exercise.

E4) Suppose that in E2, the rejected lots are screened and all defective silicon chips are replaced by non-defective ones. Calculate the average outgoing quatity (AOQ) for this plan.

So far you have learnt about the OC curve, producer's risk, consumer's risk and AOQ for double sampling plan. We now explain ASN for a double sampling plan.

8.6 AVERAGE SAMPLE NUMBER (ASN)

You have learnt in Unit 6 that the average sample number (ASN) is the expected number of sample units per lot which is required to arrive at a decision about the acceptance or rejection of the lot under the acceptance sampling plan.

In acceptance double sampling plan, the number of units inspected to arrive at a decision of acceptance or rejection of the lot depends upon whether the decision is taken only on the first sample or on the second sample as well. Therefore, we have two situations:

i) If the decision is taken on the first sample of size n_1, the number of units inspected is n_1. However, the probability of taking decision of acceptance or rejection of the lot on the first sample is P_I.

ii) If the decision is taken on the second sample of size n_2, the number of units inspected is $(n_1 + n_2)$. The probability that a second sample is necessary is $(1 - P_I)$.

Therefore, the ASN for a double sampling plan can be obtained as follows:

$$\text{ASN} = \text{Expected number of units inspected per lot}$$

$$= \sum (\text{inspected number of units} \times \text{probability of taking decision})$$

$$\text{ASN} = n_1 \times P_I + (n_1 + n_2) \times (1 - P_I) = n_1 P_I + (n_1 + n_2)(1 - P_I) \quad \ldots (20)$$

where P_I is the probability of making a decision about acceptance or rejection of the lot on the first sample and can be calculated as follows:

$$P_I = \text{P[lot is accepted on the first sample]} + \text{P[lot is rejected on the first sample]} \quad \ldots (21)$$

Let us take up an example to illustrate this concept.

Example 5: Calculate the average sample number (ASN) for Example 2.

Solution: It is given that

$$N = 400, n_1 = 15, c_1 = 1, n_2 = 30, c_2 = 3 \text{ and } p = 0.05$$

For computing the ASN, we can use equation (20):

$$ASN = n_1 P_I + (n_1 + n_2)(1 - P_I)$$

where P_I is the probability of making a decision about acceptance or rejection of the lot on the first sample. We calculate it using equation (21).

So we have to find the probability P_I first. In this double sampling plan, the lot is accepted on the first sample if $X \le c_1 = 1$, i.e., if $X \le 1$. Therefore,

$$P[\text{lot is accepted on the first sample}] = P\left[X \le c_1\right] = P\left[X \le 1\right] = P_{a1}(p)$$

We have already calculated this probability in Example 2. So we directly use the results. Therefore,

$$P_{a1}(p) = P\left[X \le c_1\right] = P\left[X \le 1\right] = 0.8290 \quad \dots \text{(i)}$$

The lot will be rejected on the first sample if $X > c_2$, i.e., $X > 3$. Therefore,

We know that $P[X > A] = 1 - P[X \le A]$

$$P[\text{lot is rejected on the first sample}] = P\left[X > 3\right] = 1 - P\left[X \le 3\right] \quad \dots \text{(ii)}$$

From Table I, for $n = n_1 = 15, x = 3$ and $p = 0.05$, we have

$$P\left[X \le 3\right] = \sum_{x=0}^{3} {}^{n_1}C_x \, p^x (1-p)^{n_1 - x} = 0.9945$$

Therefore, from equation (ii), we have

$$P[\text{lot is rejected on the first sample}] = 1 - P\left[X \le 3\right]$$

$$= 1 - 0.9945 = 0.0055 \quad \dots \text{(iii)}$$

Thus, on putting the values of equations (ii) and (iii) in equation (21), we get

$$P_I = 0.8290 + 0.0055 = 0.8345$$

Hence, on putting the values of n_1, n_2, and P_I in equation (20), we get the ASN for the plan as follows:

$$ASN = n_1 P_I + (n_1 + n_2)(1 - P_I)$$

$$= 15 \times 0.8345 + (15 + 30)(1 - 0.8345) = 19.965 \simeq 20$$

In Sec. 8.2.2, you have learnt the advantages of the double sampling plan over the single sampling plan: The average sample number (ASN) for a double sampling plan is expected to be less than that for an equivalent single sampling plan (i.e, the same probability of accepting a lot of a given quality). If we plot the ASN curves for equivalent double and single sampling plans for different values of quality level p, we obtain the curves shown in Fig. 8.2.

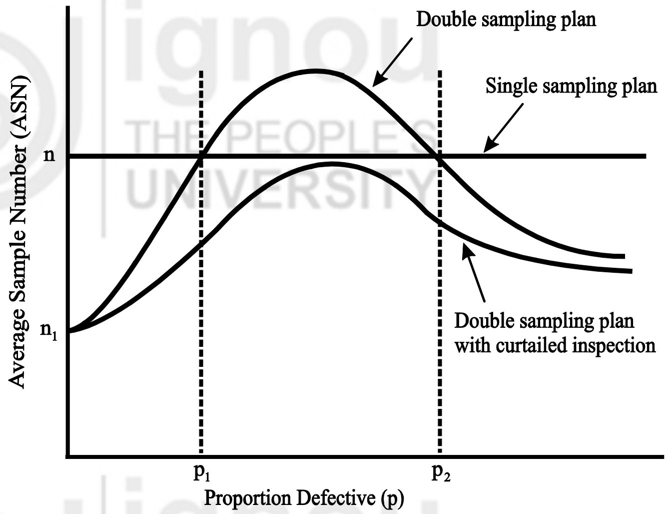

Fig. 8.2: The ASN curves for single and double sampling plans.

If we compare the ASN curves (shown in Fig. 8.2) for a double sampling plan with an equivalent single sampling plan, we observe that when the lots are of very good quality, they are usually accepted on the first sample. However, if the lots are of very bad quality, they are usually rejected on the first sample. In such cases, the ASN for the double sampling plan is smaller than the ASN for the equivalent single sampling plan because the size of the first sample in the double sampling plan is generally smaller than the size of the sample in the single sampling plan. However, if the lots are of intermediate quality (neither good nor bad), the second sample is usually required for making a decision about the acceptance or rejection of the lot in double sampling plan. In this case, the ASN for the double sampling plan is greater than the ASN for the single sampling plan.

In practice, inspection of the second sample is usually terminated and the lot is rejected as soon as the number of observed defective units in the combined samples (first and second) exceeds the acceptance number for the second sample. This termination is known as **curtailment**.

Thus, if we use the curtailed inspection for the second sample, the ASN for the double sampling plan is lower than ASN for the single sampling plan for the intermediate quality.

You can now calculate the ASN for the following exercise.

E5) Calculate the average sample number (ASN) for E2.

8.7 AVERAGE TOTAL INSPECTION (ATI)

Another important feature of the rectifying sampling plan is the average total inspection (ATI). Under rectifying sampling plan, the rejected lots are 100% inspected. The ATI is defined as follows:

The expected number of units inspected per lot under the rectifying sampling plan is called the average total inspection (ATI).

So in the rectifying double sampling plan, the number of units to be inspected will depend on the three situations given below:

i) If the lot of size N is accepted on the first sample of size n_1, the number of units inspected is n_1 and the probability of accepting the lot on the first sample is $P_{a1}(p)$.

ii) If the lot is accepted on the second sample of size n_2, the number of units inspected is $(n_1 + n_2)$ and the probability of accepting the lot on the second sample is $P_{a2}(p)$.

iii) If the lot is rejected on the first or second sample, the entire lot of size (N) is inspected and the probability of rejecting the lot is $1 - P_a(p)$.

Therefore, we can compute the ATI for a double sampling plan as follows:

$$\text{ATI} = \text{Expected number of units inspected per lot}$$

$$= \sum(\text{inspected number of units} \times \text{probability of taking decision})$$

$$\text{ATI} = n_1 P_{a1}(p) + (n_1 + n_2) P_{a2}(p) + N\left[1 - P_a(p)\right] \quad \ldots (22)$$

Let us take up an example.

Example 6: Suppose that in Example 2, the rejected lots are screened and all defective mobiles are replaced by non-defective ones. Calculate the average total inspection (ATI) for this plan.

Solution: It is given that

$$N = 400, n_1 = 15, c_1 = 1, n_2 = 30, c_2 = 3 \text{ and } p = 0.05$$

We can calculate the ATI for the double sampling plan using equation (22).

For calculating ATI, we have to calculate the probabilities of accepting the lot on the first sample and the second sample corresponding to p = 0.05.

We have already calculated these probabilities in Example 2. So we directly use the results as follows:

$$P_{a1}(p) = 0.8290, P_{a2}(p) = 0.1531 \text{ and } P_a(p) = 0.9821$$

Substituting the values of N, n_1, n_2, P_{a1} and P_{a2} in equation (22), we get

$$\text{ATI} = n_1 P_{a1}(p) + (n_1 + n_2) P_{a2}(p) + N\left[1 - P_a(p)\right]$$

$$= 15 \times 0.8290 + (15 + 30) \times 0.1531 + 400 \times (1 - 0.9821)$$

$$= 12.4350 + 6.8895 + 7.16 = 26.4845 \simeq 27$$

In the same way, you can calculate the ATI for different submitted lot qualities.

E6) Calculate the average total inspection (ATI) for E3.

We now explain how to design double sampling plans.

8.8 DESIGN OF DOUBLE SAMPLING PLANS

The design of a double sampling plan implies the determination of the parameters of the plan, i.e., the first sample size $\mathbf{n_1}$, the second sample size $\mathbf{n_2}$ and the acceptance numbers $\mathbf{c_1}$ and $\mathbf{c_2}$. These numbers have to be decided in advance before applying the double sampling plan technique. There are several approaches for determining the parameters n_1, n_2, c_1 and c_2. Here we discuss an approach for designing the double sampling plan in which the producer's risk with its corresponding acceptance quality level (AQL) and the consumer's risk with its corresponding lot tolerance percent defective (LTPD) are specified.

We have to design a double sampling plan, which satisfies both the producer's and the consumer's risk, such that the lots of AQL are to be rejected no more than 100α% of the time and lots of LTPD are to be accepted no more than 100β% of the time.

For designing the plan in such situations, we use a pair of tables known as Grubbs Tables (Tables IV and V given at the end of this block). Tables IV and V are based on a relationship imposed on the parameters $\mathbf{n_1}$ and $\mathbf{n_2}$, that $\mathbf{n_1 = n_2}$ and $\mathbf{n_1 = 2n_2}$, respectively. Both tables are based on a producer's risk (α) of 0.05 and a consumer's risk (β) of 0.10. Table IV is used when $\mathbf{n_1 = n_2}$ and Table V is used when $\mathbf{n_1 = 2n_2}$. In this approach, we first find the operating ratio R as follows:

$$R = \frac{p_2}{p_1} \qquad \ldots (23)$$

The values of R for double sampling plan are also listed in Table IV or V. We choose a value of R which is exactly equal to its desired value. Generally, the tabulated value of R is not equal to the desired value of R. In such situations, we take the value closest to the calculated value of R. Then we look up the corresponding values of c_1, c_2 and n_1p from the appropriate table. There are two columns for n_1p, one for α = 0.05 and another for β = 0.10.

The following two approaches are used for deciding the value of n_1:

1. **Satisfy Producer's Risk Stipulation Exactly and come close to Consumer's Risk**

   According to this approach, we look up the value of n_1p for α = 0.05. Then we obtain the value of n_1 by dividing n_1p by $p = p_1$ = AQL as follows:

$$n_1 = \frac{n_1 p}{p_1} \qquad \ldots (24)$$

2. **Satisfy Consumer's Risk Stipulation Exactly and come close to Producer's Risk**

   According to this approach, we look up the value of n_1p for β = 0.10. Then we obtain the value of n_1 by dividing n_1p by $p = p_2$ = LTPD as follows:

$$n_1 = \frac{n_1 p}{p_2} \qquad \ldots (25)$$

If the computed value of n_1 is a fraction, it is rounded off to the next integer.

We then obtain the value of n_2 by taking $n_1 = n_2$ or $n_1 = 2n_2$ as the case may be.

Let us consider an example to illustrate how to design a specific double sampling plan.

Example 7: Suppose a tyre supplier ships tyres in lots of size 500 to the buyer. The supplier and the quality control inspector of the buyer decide the acceptance quality level (AQL) to be 2% and the lot tolerance percent defective (LTPD) to be 8%. Design a double sampling plan which ensures that lots of quality 2% will be rejected about 5% of the time and lots of quality 8% will be accepted about 10% of the time. The sample sizes are equal.

Solution: It is given that

$$AQL = p_1 = 2\% = 0.02 \text{ and } \alpha = 5\% = 0.05$$

$$LTPD = p_2 = 8\% = 0.08 \text{ and } \beta = 10\% = 0.10$$

$$n_1 = n_2$$

To design the desired sampling plan, we first calculate the operating ratio (R) from equation (23) as follows:

$$R = \frac{p_2}{p_1} = \frac{0.08}{0.02} = 4.0$$

Since sample sizes are equal, we use Table IV to look up the value of R. We see from Table IV that the desired value of R = 4.0 lies closest to 3.88. Then we look up the corresponding value of c_1 and c_2. From Table IV, we have $c_1 = 2$ and $c_2 = 5$. We have to find the value of the first sample size (n_1).

We first find the plan which satisfies the desired producer's risk exactly.

For this we look up the value of n_1p corresponding to $c_1 = 2$, $c_2 = 5$ and $\alpha = 0.05$ in Table IV.

From Table IV, we have $n_1p = 1.43$. Therefore, from equation (24), we have

$$n_1 = \frac{n_1p}{p_1} = \frac{1.43}{0.02} = 71.5 \simeq 72$$

Hence, the required double sampling plan is

$$n_1 = 72,\ n_2 = 72, c_1 = 2 \text{ and } c_2 = 5$$

We now find the plan which satisfies the desired consumer's risk exactly.

From Table IV, we have $n_1p = 5.55$ corresponding to $c_1 = 2$, $c_2 = 5$ and $\beta = 0.10$. Therefore, from equation (25), we have

$$n_1 = \frac{n_1p}{p_2} = \frac{5.55}{0.10} = 55.5 \simeq 56$$

Hence, the required double sampling plan is

$$n_1 = 56,\ n_2 = 56, c_1 = 2 \text{ and } c_2 = 5$$

You may like to design a sampling plan yourself. Try the following exercise.

E7) A computer manufacturer purchases a computer part from a supplier in lots of 2000. The supplier and the quality control inspector of the company decide the acceptance quality level (AQL) to be 1.5% and the lot tolerance percent defective (LTPD) to be 10%. Assuming that the lot of the second sample is twice that of the first sample, design a double sampling plan which ensures that lots of quality 1.5% will be rejected about 5% of the time and lots of quality 10% will be accepted about 10% of the time.

We now end this unit by giving a summary of what we have covered in it.

8.9 SUMMARY

1. A sampling plan in which a decision about the acceptance or rejection of a lot is based on the inspection of two samples is known as a **double sampling plan**. There are four parameters of a double sampling plan:

n_1 – size of the first sample,

c_1 – acceptance number for the first sample,

n_2 – size of the second sample, and

c_2 – acceptance numbers for both samples combined.

2. The probability of accepting a lot of quality p under a double sampling plan is given by

$$P_a(p) = \sum_{x=0}^{c_1} {}^{n_1}C_x\, p^x (1-p)^{n_1-x}$$
$$+ \sum_{x=c_1+1}^{c_2} \sum_{y=0}^{c_2-x} {}^{n_1}C_x\, p^x (1-p)^{n_1-x}\; {}^{n_2}C_y\, p^y (1-p)^{n_2-y}$$

3. The AOQ for a double sampling plan is

$$AOQ = p\left[\left(1-\frac{n_1}{N}\right)(P_{a1}+P_{a2}) - \frac{n_2}{N}P_{a2}\right]$$

4. The ASN for a double sampling plan is

$$ASN = n_1P_I + (n_1+n_2)(1-P_I)$$

where P_I is the probability of making a decision about the acceptance or rejection of the lot on the first sample and is given by

$$P_I = P[\text{lot is accepted on the first sample}] + P[\text{lot is rejected on the first sample}]$$

5. The ATI for a double sampling plan is

$$ATI = n_1P_{a1} + (n_1+n_2)P_{a2} + N[1-P_a]$$

8.10 SOLUTIONS/ANSWERS

E1) For implementing the double sampling plan, the buyer randomly draws the first sample of 5 chips from the lot and classifies each chip of the first sample as defective or non-defective. At the end of the inspection, he/she counts the number of defective chips (d_1) found in the first sample and compares the number of defective chips (d_1) with the acceptance numbers c_1 and c_2. If $d_1 \le c_1 = 0$, he/she accepts the lot and if $d_1 > c_2 = 2$, he/she rejects the lot. If $c_1 < d_1 \le c_2$, it means that if the number of defective chips is 1, he /she draws the second sample of size 20 from the lot. He/she counts the number of defective chips (d_2) found in the second sample and compares the total number of defective chips ($d_1 + d_2$) in both samples with the acceptance numbers c_2. If $d_1 + d_2 \le c_2 = 2$, he/she accepts the lot and if $d_1 + d_2 > c_2 = 2$, he/she rejects the lot.

E2) It is given that

$$N = 1000, n_1 = 5, c_1 = 0, n_2 = 20, c_2 = 2 \text{ and } p = 0.03$$

If $P_{a1}(p)$ and $P_{a2}(p)$ denote the probabilities of accepting the lot on the first sample and the second sample, respectively, we can calculate the probability of accepting the lot of quality level p in a double sampling plan as follows:

$$P_a(p) = P_{a1}(p) + P_{a2}(p) \qquad \ldots \text{(i)}$$

If X represents the number of defective chips in the first sample of size n_1, the lot is accepted on the first sample if $X \le c_1 = 0$. Therefore,

$$P_{a1}(p) = P[X \le c_1] = P[X \le 0] \quad \ldots \text{(ii)}$$

Since $N \ge 10n_1$, we can use the binomial distribution and calculate $P_{a1}(p)$ using Table I.

From Table I, for $n = n_1 = 5, x = c_1 = 0$ and $p = p_1 = 0.03$, we have

$$P_{a1}(p) = P[X \le 0] = \sum_{x=0}^{0} {}^{n_1}C_x\, p^x (1-p)^{n_1 - x} = 0.8587 \quad \ldots \text{(iii)}$$

Hence, the probability of accepting the lot on the first sample is 0.8587. It means that about 85.87% of the lot of quality p = 0.03 will be accepted on the first sample.

If Y represents the number of defective chips in the second sample, the lot is accepted on the second sample if $c_1 < X + Y \le c_2$. It means that we shall accept the lot if

$X = c_1 + 1 = 0 + 1 = 1$ and $Y \le c_2 - X = 2 - 1 = 1$ or

$X = c_1 + 2 = 0 + 2 = 2$ and $Y \le c_2 - X = 2 - 2 = 0$.

Therefore,

$$P_{a2}(p) = P[X = 1]P[Y \le 1] + P[X = 2]P[Y \le 0] \quad \ldots \text{(iv)}$$

We also know that $P[X = x] = P[X \le x] - P[X \le x - 1]$

From Table I, for $n = n_1 = 5$ and $p = 0.03$, we have

$$P[X = 1] = P[X \le 1] - P[X \le 0] = 0.9915 - 0.8587 = 0.1328$$

$$P[X = 2] = P[X \le 2] - P[X \le 1] = 0.9997 - 0.9915 = 0.0082$$

From Table I, for $n = n_2 = 20$ and $p = 0.03$, we have

$$P[Y \le 1] = 0.8802 \text{ and } P[Y \le 0] = 0.5438$$

On putting these values in equation (iv), we get

$$P_{a2}(p) = P[X = 1]P[Y \le 1] + P[X = 2]P[Y \le 0]$$

$$= 0.1328 \times 0.8802 + 0.008 \times 0.5438 = 0.1213$$

Hence, the probability of accepting the lot on the second sample is 0.1213. It means that about 12.13% of the lot of quality p = 0.03 will be accepted on the second sample.

Thus, from equation (i), we get the probability of accepting a lot in a double sampling plan as follows:

$$P_a(p) = P_{a1}(p) + P_{a2}(p) = 0.8587 + 0.1213 = 0.9800$$

Thus, overall, 98% of the lots will be accepted by this sampling plan.

E3) It is given that

$$N = 1000, n_1 = 5, c_1 = 0, n_2 = 20, c_2 = 2,$$

$AQL = p_1 = 0.05$ and $LTPD = p_2 = 0.14$

For calculating the producer's risk, we can use equation (11):

$$P_p = P[\text{rejecting a lot of acceptance quality level } p_1]$$

$$= 1 - P_a(p_1)$$

Therefore, for calculating the producer's risk, we first calculate the probability of accepting the lot of quality $p = p_1 = AQL = 0.05$.

In a double sampling plan, the lot may be accepted either on the first sample or on the second sample. So if $P_{a1}(p)$ and $P_{a2}(p)$ denote the probabilities of accepting the lot on the first sample and the second sample, respectively, we can calculate the probability of acceptance of the lot of quality level p as follows:

$$P_a(p) = P_{a1}(p) + P_{a2}(p) \quad \ldots \text{(i)}$$

If X represents the number of defective chips in the first sample, the lot will be accepted on the first sample if $X \le c_1$, i.e., $X \le 0$. Therefore,

$$P_{a1}(p) = P[X \le c] = P[X \le 0] \quad \ldots \text{(ii)}$$

Since $N \ge 10n_1$, we can use the binomial distribution and calculate the probability using Table I.

From Table I, for $n = n_1 = 5, x = c_1 = 0$ and $p = p_1 = 0.05$, we have

$$P_{a1}(p_1) = P[X \le 0] = \sum_{x=0}^{0} {}^{n_1}C_x\, p_1^x (1-p_1)^{n_1 - x} = 0.7738 \quad \ldots \text{(iii)}$$

If Y represents the number of defective chips in the second sample, the lot will be accepted on the second sample if $c_1 < X + Y \le c_2$. It means that we shall accept the lot if

$X = c_1 + 1 = 0 + 1 = 1$ and $Y \le c_2 - X = 2 - 1 = 1$

or $X = c_1 + 2 = 0 + 2 = 2$ and $Y \le c_2 - X = 2 - 2 = 0$.

Therefore,

$$P_{a2}(p_2) = P[X = 1]P[Y \le 1] + P[X = 2]P[Y \le 0] \quad \ldots \text{(iv)}$$

We also know that $P[X = x] = P[X \le x] - P[X \le x - 1]$

From Table I, for $n = n_1 = 5$ and $p = p_1 = 0.05$, we have

$$P[X = 1] = P[X \le 1] - P[X \le 0] = 0.9774 - 0.7738 = 0.2036$$

$$P[X = 2] = P[X \le 2] - P[X \le 1] = 0.9988 - 0.9774 = 0.0214$$

From Table I, for $n = n_2 = 20$ and $p = p_1 = 0.05$, we have

$$P[Y \le 0] = 0.3585 \text{ and } P[Y \le 1] = 0.7358$$

On putting these values in equation (iv), we get

$$P_{a2}(p_2) = P[X = 1]P[Y \le 1] + P[X = 2]P[Y \le 0]$$

$$= 0.2036 \times 0.7358 + 0.0214 \times 0.3585 = 0.1575$$

Hence, from equation (i), we get the probability of accepting the lot of quality $p = p_1 = 0.05$ in this double sampling plan as follows:

$$P_a(p_1) = P_{a1}(p_1) + P_{a2}(p_1) = 0.7738 + 0.1575 = 0.9313$$

Therefore, we calculate the producer's risk for this plan using equation (11) as follows:

$$P_p = 1 - P_a(p_1) = 1 - 0.9313 = 0.0687$$

Similarly, we can calculate the consumer's risk as follows:

We can calculate the consumer's risk for the double sampling plan using equation (12) which is given by

$$P_c(p) = P[\text{accepting a lot of quality (LTPD)} \, p_2] = P_a(p_2) \qquad \ldots \text{(v)}$$

We first calculate the probability of accepting the lot of quality $p = p_1 = \text{LTPD} = 0.14$ using equations (i), (ii) and (iv).

From Table I, for $n = n_1 = 5, x = c_1 = 0$ and $p = p_2 = 0.14$, we have

$$P_{a1}(p_2) = P[X \le 0] = \sum_{x=0}^{0} {}^{n_1}C_x \, p_2^x (1 - p_2)^{n_1 - x} = 0.4704 \qquad \ldots \text{(vi)}$$

From Table I, for $n = n_1 = 5$ and $p = p_2 = 0.14$, we have

$$P[X = 1] = P[X \le 1] - P[X \le 0] = 0.8533 - 0.4704 = 0.3829$$

$$P[X = 2] = P[X \le 2] - P[X \le 1] = 0.9780 - 0.8533 = 0.1247$$

From Table I, for $n = n_2 = 20$ and $p = p_2 = 0.14$, we have

$$P[Y \le 0] = 0.0490 \text{ and } P[Y \le 1] = 0.2084$$

On putting these values in equation (iv), we get

$$P_{a2}(p_2) = P[X = 1]P[Y \le 1] + P[X = 2]P[Y \le 0]$$

$$= 0.3829 \times 0.2084 + 0.1247 \times 0.0490 = 0.0859 \qquad \ldots \text{(vii)}$$

On putting the values of $P_{a1}(p_2)$ and $P_{a2}(p_2)$ in equation (i), we get

$$P_a(p_2) = 0.4704 + 0.0859 = 0.5563$$

Hence, from equation (v), we get, the consumer's risk for this plan as follows:

$$P_c = P_a(p_2) = 0.5563$$

E4) It is given that

$$N = 1000, n_1 = 5, c_1 = 0, n_2 = 20, c_2 = 2 \text{ and } p = 0.03$$

It is noted that the sample sizes $n_1 = 5$ and $n_2 = 20$ are very small in proportion to the lot size N = 1000, i.e., $n_1 / N \approx 0$ and $n_2 / N \approx 0$. So we can calculate the average outgoing quality (AOQ) for the double sampling plan using equation (19).

For calculating AOQ, we have to calculate the probabilities of accepting the lot on the first sample and the second sample corresponding to $p = 0.03$.

We have already calculated these probabilities in E2. So we directly use the results:

$$P_{a1}(p) = 0.8587 \text{ and } P_{a2}(p) = 0.1213$$

Substituting the values of P_{a1}, P_{a2} and p in equation (19), we get

$$AOQ = p(P_{a1} + P_{a2}) = 0.03 \times (0.8587 + 0.1213) = 0.0294$$

E5) It is given that

$$N = 1000, n_1 = 5, c_1 = 0, n_2 = 20, c_2 = 2 \text{ and } p = 0.03$$

For computing ASN for the double sampling plan, we can use equation (20):

$$ASN = n_1 P_I + (n_1 + n_2)(1 - P_I)$$

where P_I is the probability of making a decision about the acceptance or rejection of the lot on the first sample and can be calculated as follows:

$$P_I = P[\text{lot is accepted on the first sample}] + P[\text{lot is rejected on the first sample}] \quad \ldots \text{(i)}$$

So we have to find the probability P_I first. In the double sampling plan, the lot is accepted on the first sample if $X \le c_1$, i.e., if $X \le 0$. Therefore,

$$P[\text{lot is accepted on the first sample}] = P[X \le c_1] = P[X \le 0] = P_{a1}$$

We have already calculated this probability in E2. So we directly use the results. Therefore,

$$P_{a1}(p) = P[X \le 0] = 0.8587 \quad \ldots \text{(ii)}$$

The lot will be rejected on the first sample if $X > c_2$, i.e., if $X > 2$. Therefore,

$$P[\text{lot is rejected on the first sample}] = P[X > 2] = 1 - P[X \le 2] \quad \ldots \text{(iii)}$$

From Table I, for $n = n_1 = 5, x = c_2 = 2$ and $p = 0.03$, we have

$$P[X \le 2] = \sum_{x=0}^{2} {}^{n_1}C_x\, p^x (1-p)^{n_1 - x} = 0.9997$$

Therefore, from equation (iii), we have

$$P[\text{lot is rejected on the first sample}] = 1 - P[X \le 2] = 1 - 0.9997 = 0.0003 \quad \ldots \text{(iv)}$$

Thus, on putting the values from equations (ii) and (iv) in equation (i), we get

$$P_I = 0.8587 + 0.0003 = 0.8590$$

Hence, on putting the values of n_1, n_2, and P_I in equation (20), we get the ASN for the plan as follows:

$$ASN = n_1P_I + (n_1 + n_2)(1 - P_I)$$

$$= 5 \times 0.8590 + (5 + 20)(1 - 0.8590) = 7.82 \approx 8$$

E6) It is given that

$$N = 1000, n_1 = 5, c_1 = 0, n_2 = 20, c_2 = 2 \text{ and } p = 0.03$$

We can calculate the ATI for the double sampling plan using equation (22).

For calculating ATI, we have to calculate the probabilities of accepting the lot on the first sample and the second sample corresponding to $p = 0.03$.

We have already calculated these probabilities in E2. So we directly use the results:

$$P_{a1}(p) = 0.8587, P_{a2}(p) = 0.1213 \text{ and } P_a(p) = 0.9800$$

Substituting the values of N, n_1, n_2, P_{a1} and P_{a2} in equation (22), we get

$$ATI = n_1P_{a1}(p) + (n_1 + n_2)P_{a2}(p) + N[1 - P_a(p)]$$

$$= 5 \times 0.8587 + (5 + 20) \times 0.1213 + 1000 \times (1 - 0.9800)$$

$$= 4.2935 + 3.0325 + 20 = 27.3260 \approx 28$$

E7) It is given that

$$AQL = p_1 = 1.5\% = 0.015 \text{ and } \alpha = 5\% = 0.05$$

$$LTPD = p_2 = 10\% = 0.10 \text{ and } \beta = 10\% = 0.10$$

$$n_2 = 2n_1$$

To design the desired sampling plan, we first calculate the operating ratio (R) from equation (23) as follows:

$$R = \frac{p_2}{p_1} = \frac{0.10}{0.015} = 6.67$$

Since $n_2 = 2n_1$, we use Table V to look up the value of R. We see from Table V that the desired value of R = 6.67 lies closest to 6.48. Then we look up the corresponding values of c_1 and c_2. From Table V, we have $c_1 = 1$ and $c_2 = 3$. We have to find the value of the first sample size (n_1).

We first find the plan which satisfies the desired producer's risk exactly.

For this we look up the value of n_1p corresponding $c_1 = 1$, $c_2 = 3$ and $\alpha = 0.05$ in Table V.

From Table V, we have $n_1p = 0.60$. Therefore, from equation (24), we have

$$n_1 = \frac{n_1p}{p_1} = \frac{0.60}{0.015} = 40$$

Hence, the required double sampling plan is

$$n_1 = 40, n_2 = 80, c_1 = 1 \text{ and } c_2 = 3$$

We now find the plan which satisfies the desired consumer's risk exactly.

From Table V, we have $n_1p = 3.89$ corresponding to $c_1 = 1$, $c_2 = 3$ and $\beta = 0.10$. Therefore, from equation (25), we have

$$n_1 = \frac{n_1 p}{p_2} = \frac{3.89}{0.10} = 38.9 \simeq 39$$

Hence, the required double sampling plan is

$$n_1 = 39,\ n_2 = 78, c_1 = 1 \text{ and } c_2 = 3$$